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**Chapter _____: _____________________________________________________**

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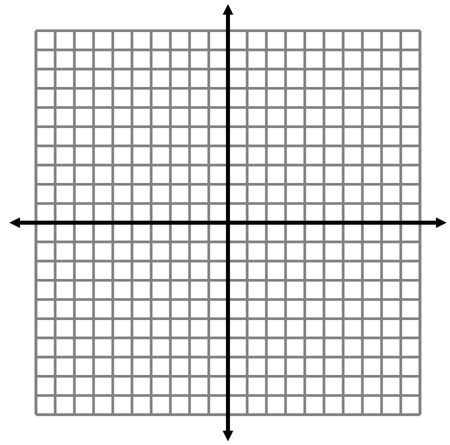
# PARABOLA INVESTIGATION

Graph $y = x^2$ by completing the chart below.

| x | y |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |



What shape is the graph?_____________________________________________________All parabola graphs are the same shape. The changes that occur to the graph include the position of the vertex of the graph, the location of the axis of symmetry, whether the graph is “wide” or “narrow”, and whether the graph is concave up or concave down.

---

The following is an investigation on graphing parabolas using the graphing calculator. When you have completed this investigation, you should be able to completely describe and analyze the equation of any parabola written in the form:

$$y = a(x-h)^2 + k$$

You should be able to describe completely what the letters ***a***, ***h***, and ***k*** do to the parabola in the equation $y = a(x-h)^2 + k$ .

Enter the functions in the **y=** menu one at a time. Display $y_1$ before you enter $y_2$. Display $y_1$ and $y_2$ before you enter $y_3$. Display all three functions after you enter $y_3$. Answer the questions as you go along. Be careful with the parentheses.
(Use the following window settings: $X\min = -11.75$ $X\max = 11.75$ $Xscl = 1$ $Y\min = -7.75$ $Y\max = 7.75$ $Yscl = 1$)

---

$y_1 = x^2$
$y_2 = 3x^2$ How has the graph changed?_______________________________________________
$y_3 = (1/5)x^2$ How has the graph changed?_______________________________________________

How would the graph of $y = 4x^2$ compare to the graph of $y = x^2$?________________________________

How would the graph of $y = (2/3)x^2$ compare to the graph of $y = x^2$?____________________________

Graph the equations on the calculator and check your answers.

---

Clear out the three functions and enter the following three functions in the same manner as above. Answer the questions that follow.
$y_1 = x^2$
$y_2 = -x^2$
$y_3 = -3x^2$

What did the –1 and –3 do to the graph of $y = x^2$?__________________________________________
In general, what does the letter **a** do in the graph $y = ax^2$ ? (Be sure to include in your description whether **a** is positive or negative.____________________________________________________________________
______________________________________________________________________________

over→

Clear out the three functions and enter the following three functions in the same manner as above. Answer the questions that follow.

$y_1 = x^2$
$y_2 = x^2 + 5$
$y_3 = x^2 - 2$

What did the 5 and –2 do to the vertex of $y = x^2$?________________________________________

In general, what will the letter **k** in $y = x^2 + k$ do to the vertex the graph of $y = x^2$? ________________________________

____________________________________________________________________________

---

Clear out the three functions and enter the following three functions in the same manner as before. Answer the questions that follow.

$y_1 = x^2$
$y_2 = (x+3)^2$
$y_3 = (x-5)^2$

What did the 3 and –5 do to the vertex of $y = x^2$?________________________________________

In general, what will the letter **h** in $y = (x-h)^2$ do to the vertex the graph of $y = x^2$? ______________________________

____________________________________________________________________________

---

What will be the coordinates of the vertex of $y = (x-4)^2 + 3$?____________________

Graph the equation and see if you are correct.

What will be the coordinates of the vertex of $y = (x-1)^2 - 6$?____________________

Graph the equation and see if you are correct.

---

Describe the graph of each of the following using the words narrow or wide and concave up or concave down. Tell where the vertex is for each graph and what the axis of symmetry is. After describing each one, graph it on the graphing calculator and see if you are correct.

$y = -4(x-6)^2 + 2$

$y = (2/3)(x+4)^2 - 1$

$y = 2(x-1.5)^2 + 3.25$

$y = -3(x-5)^2 + 8$

---

Describe completely what the letters ***a, h***, and ***k*** do to the parabola in the equation $y = a(x-h)^2 + k$.

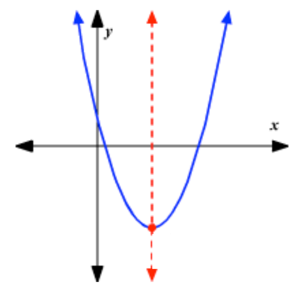
## 5.1 Graphing Quadratic Functions

A **quadratic function** is described by an equation of the following form:

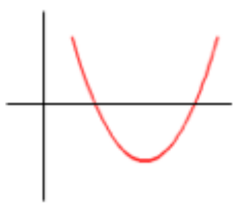
**Graph of a Quadratic Equation**

Consider the graph of ____________________________, where a $\neq 0$.

- The **y-intercept** is ___________________ or ______.
- The **equation of the axis of symmetry** is _________________.
- The **x-coordinate of the vertex** is _________________.

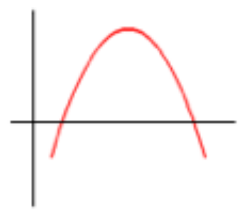



**Minimum and Maximum Value**



**Minimum**

a is ______________
and the **range** is all real numbers greater than or equal to the minimum



**Maximum**

a is ______________
and the **range** is all real numbers less than or equal to the maximum

**Example 1**: **Graph of a Quadratic Function**
Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function, **and graph**.

a) $f(x) = x^2 - 8x + 2$

Maximum or minimum? _________

Value? _________

Domain? ______________________

Range? _______________________

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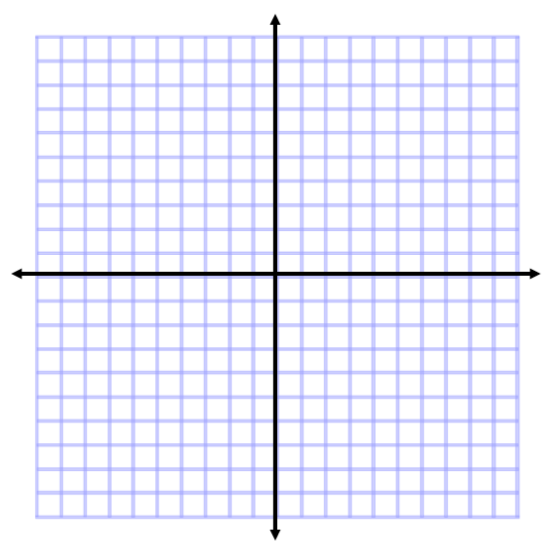

b) $f(x) = 4x - x^2 + 1$

Maximum or minimum? ________

Value? ________

Domain? ____________________

Range? _____________________

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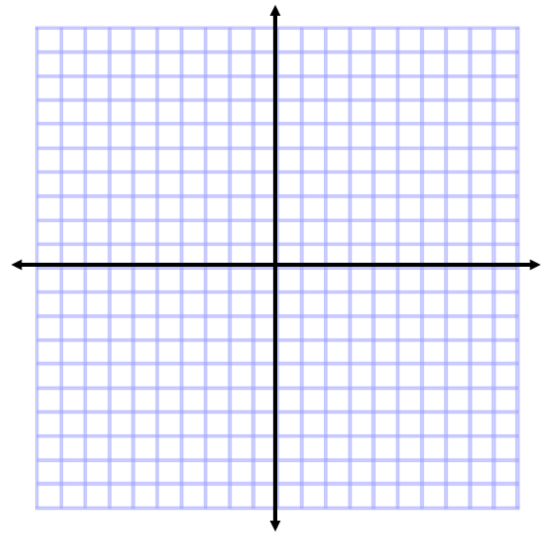


**Example 2: Real-World Situations**

a) From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height *h(t)* of the ball *t* seconds after Susan throws it is given by the equation below. Find the maximum height reached by the ball and the time that this height is reached.

$$h(t) = -16t^2 + 32t + 4$$

Maximum height reached by the ball: __________ Time that the height is reached: __________

b) Consider the following problem: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river. What are the dimensions of the field of largest area that he can fence?

Dimensions of the field of largest area that he can fence: ____________________

**5.2 Solving Quadratic Equations by Graphing**

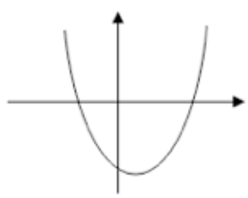
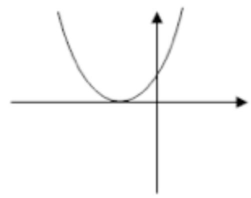
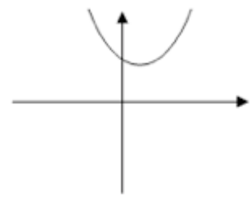
A **quadratic equation** can be written in the form:

When a quadratic equation is written in this way, and *a, b,* and *c* are all integers, it is in ________________.

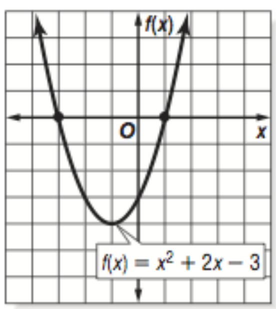
**Solutions of a Quadratic Equation**
There are many terms associated with the solution of a quadratic equation. They are:

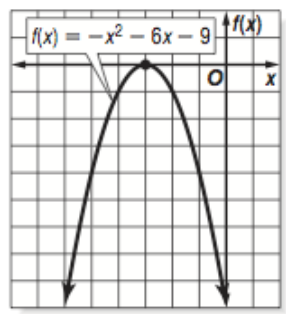
1. __________________________ 2. __________________________ 3. __________________________

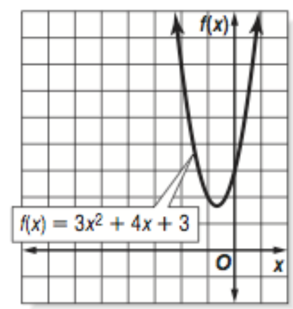
A quadratic equation can have **two real solutions**, **one real solution,** or **no real solution**.

**Example 1: Determining Solutions**
Use the related graph of each equation to determine its solutions.

a)


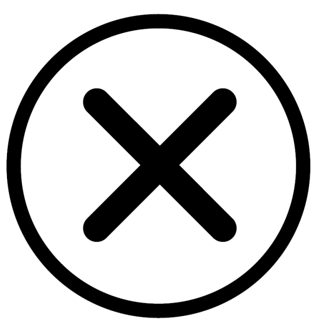

b)



c)



solution(s): ________________ solution(s): ________________ solution(s): ________________

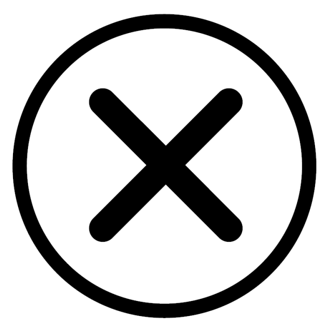
**Example 2: Number Theory**
Find two real numbers that satisfy each situation, or show that no such numbers exist.

a) Their sum is -4, and their product is 0



numbers: __________________________

b) Their sum is 0, and their product is -36



numbers: __________________________

My teacher says that fountains produce parabolic streams.

1. Plot the points of this fountain and see if you can deduce the equation. Excel helps a ton with this.

| X | Y |
|---|---|
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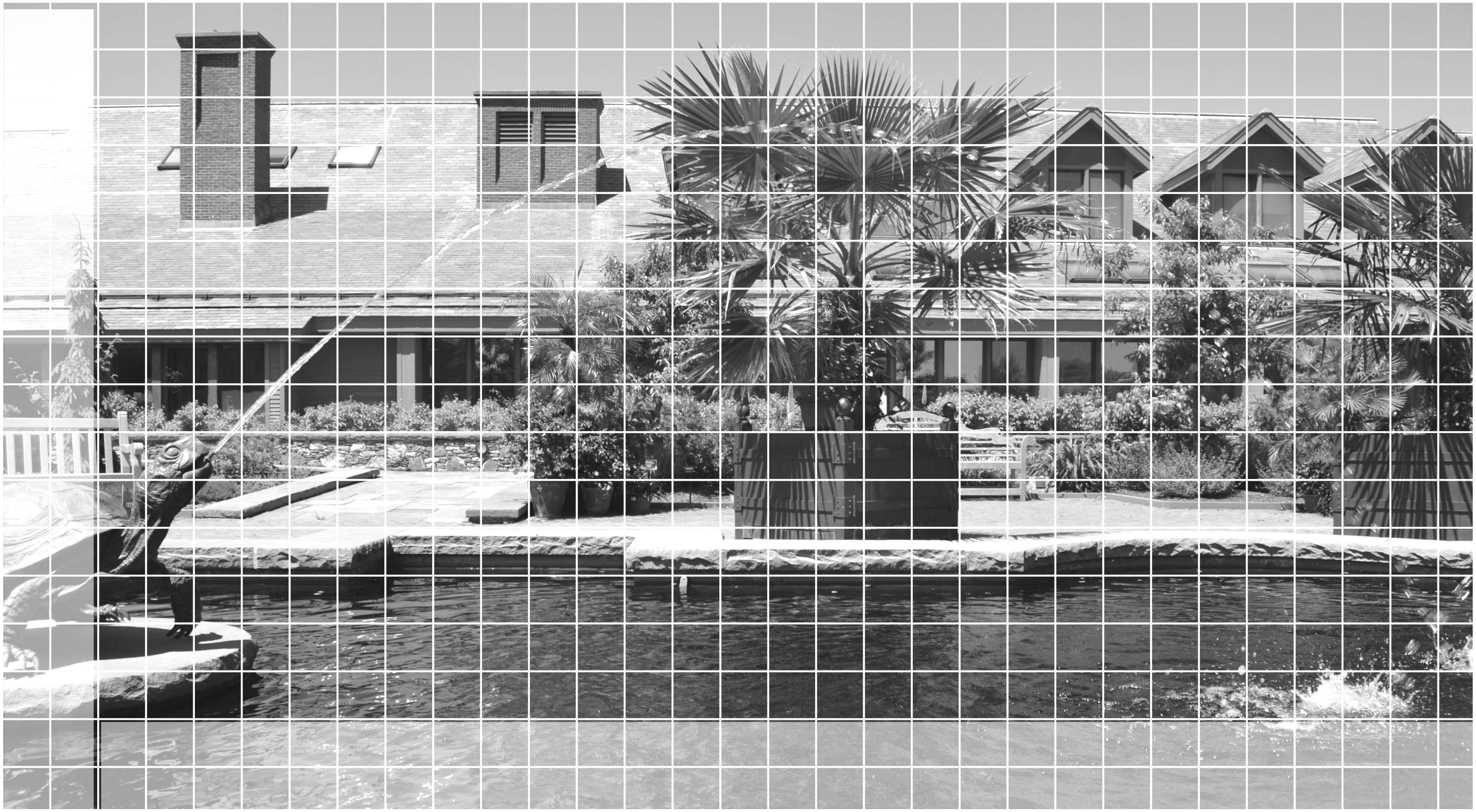


My teacher also says that you can tell the highest point of the stream by arranging the parabolic equation in this way,

$Y = a(X - h)^2 + k$ where the vertex of the parabola is at (h , k)

2. Try to arrange your equation in this form and then look back at the picture to see where the high point of the stream is.

3. Was my teacher correct?

## 6.1 Properties of Exponents

| **Name of Property** | **Formula** |
| --- | --- |
| 1. ____________________ | ____________________ |
| 2. ____________________ | ____________________ |
| 3. ____________________ | ____________________ |
| 4. ____________________ | ____________________ |
| 5. ____________________ | ____________________ |
| 6. ____________________ | ____________________ |

**Simplifying Expressions**
A monomial expression is in simplified form when:

1. ________________________________________
2. ________________________________________
3. ________________________________________

**Examples: Simplifying Expressions Using Several Properties**
Simplify. Assume that no variable equals 0.

a) $(-5x^4y^3)(-3xy^5)$

b) $8u(2z)^3$

c) $(-3c^2d^5)^3$

d) $\left(\frac{-2a}{b^2}\right)^5$

e) $\frac{-6s^5x^3}{18sx^7}$

f) $\frac{-27x^3(-x^7)}{16x^4}$

g) $(m^4n^6)^4(m^3n^2p^5)^6$

## 6.2 Operations with Polynomials

**Example 1: Simplify Polynomials**

a) $(-x^2 - 3x + 4) - (x^2 + 2x + 5)$

b) $(3x^2 - 6) + (-x + 1)$

**Example 2: Simplify Using the Distributive Property**

a) $-2a(-3a^2 - 11a + 20)$

b) $-y(4y^2 + 2y - 3)$

**Example 3: Multiply Polynomials**

a) $(v^2 - 6)(v^2 + 4)$

b) $(y - 8)^2$

c) $(x^2 + 4x + 16)(x - 4)$

**Example 4: Real-World Situations**

Walter Waterman, of Walter's Water Pumps in Winnipeg has found that when he produces *x* water pumps per month, his revenue is $x^2 + 400x + 300$ dollars. His cost for producing *x* water pumps per month is $x^2 + 300x - 200$ dollars.

a) Write a polynomial to represent his monthly profit for *x* pumps.

b) Find the profit from sales of 50 water pumps.

## Factoring Review (Day One)

**Factoring Techniques** | **When Do You Use It?**

1. ______________________ **# of terms:** ______________________

**Example:** ______________________

2. ______________________ **# of terms:** ______________________

**Example:** ______________________

3. ______________________ **# of terms:** ______________________

**Example:** ______________________

**REMINDER!** ______________________

______________________

4. ______________________ **# of terms:** ______________________

**Example:** ______________________

**REMINDER!** ______________________

______________________

______________________

**Examples: Factoring Polynomials Completely**

a) $6xyz + 8xy^2$

b) $x^2 - 49$

c) $x^2 + 8x + 12$

d) $x^2 + 14x + 49$

e) $36x^2 - 49y^2$

f) $3x^2 - 15x - 42$

g) $9x^2 - 24x + 16$

h) $4x^2 - 36$

## Factoring Review (Day Two)

| **Factoring Techniques** | **When Do You Use It?** | |
| --- | --- | --- |
| 1. ______________________ | **# of terms:** | ______________________ |
| | **Example:** | ______________________ |
| 2. ______________________ | **# of terms:** | ______________________ |
| | **Example:** | ______________________ |
| | **REMINDER!** | ______________________ |
| | | ______________________ |
| 3. ______________________ | **# of terms:** | ______________________ |
| | **Example:** | ______________________ |
| | **REMINDER!** | ______________________ |
| | | ______________________ |

**Examples: Factoring Polynomials Completely**

a) $6x^2 - 7x - 10$

b) $x^3 - 64$

c) $8x^3 + 125y^3$

d) $15x^2 + 13x + 2$

e) $2x^2 + 7x - 9$

f) $3x^4 + 81x$

## FACTORING WORKSHEET #1

Factor the following completely.

1. $6a + 6b$
2. $y^3 - y^2$
3. $x^2 + xy + 3x$
4. $8m^2 + 16am + 8m$
5. $b^2 - 144$
6. $16 - x^2$
7. $4a^2 - 9$
8. $25x^2 - 16y^2$
9. $a^2 + 12a + 35$
10. $d^2 + 4d - 21$
11. $b^2 + 7b + 6$
12. $b^2 - b - 6$
13. $k^2 + 12k + 36$
14. $w^2 - 8w + 16$
15. $9a^2 - 12ab + 4b^2$
16. $16x^2 + 40x + 25$

Factor the following completely. You may need to use more than one method. If the polynomial is not factorable, write PRIME.

17. $3x^2 + 21x + 36$
18. $x^4 - 16$
19. $18g^2 - 24g + 8$
20. $2y^3 - 8y^2 - 42y$
21. $x^3 + 2x^2 - 35x$
22. $xy^2 - x^2y$
23. $27p^2 - 12q^2$
24. $x^2 + 4x + 5$
25. $9x^2 - 24xy + 16y^2$
26. $r^4 + r^3s + r^2s^2$
27. $81x^2 + 49$
28. $3x^2 + 6x + 9$
29. $16x^2 - 9y^2$
30. $64x^2 + 80xy + 25y^2$
31. $2x^2 - 26x + 60$
32. $2x^2 - 26x - 60$

---

## FACTORING WORKSHEET #2

Factor the following completely.

1. $3x^2 + 5x + 2$
2. $4x^2 + 11x + 6$
3. $2a^2 - 7a - 15$
4. $6x^2 + 13x + 6$
5. $5x^2 - 4x - 9$
6. $9b^2 - 18b - 16$
7. $4x^2 - 20x + 25$
8. $18x^2 + 3xy - 10y^2$
9. $x^3 - 8$
10. $27 + y^3$
11. $8x^3 + 1$
12. $x^3 - 125$
13. $64x^3 - 27$
14. $x^6 + 8$
15. $a^3b^3 - 64$
16. $x^9 - y^6$

Factor the following completely. You may need to use more than one method. If the polynomial is not factorable, write PRIME.

17. $6a^2 + 27a - 15$
18. $2r^3 - 16s^3$
19. $x^3 + 64$
20. $x^2 + 64$
21. $x^2 - 64$
22. $4x^2 + 8x - 96$
23. $2b^2 + 13b - 7$
24. $2x^2 + 7x + 1$
25. $9x^2 - 30x + 25$
26. $3x^3 - 24$
27. $8m^3 - 25$
28. $x^6 - 64$

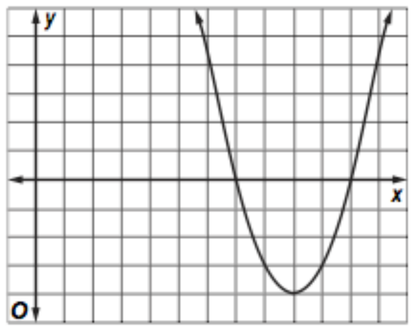
## 5.3 Solving Quadratic Equations by Factoring

**Intercept Form of a Quadratic Equation**

Any function in **intercept form** can be transformed to **standard form** by using the **FOIL method** for multiplying binomials.

**Example 1: Write an Equation Given Roots**

a) Write a quadratic equation in standard form for the following graph:




b) Write a quadratic equation in standard form with the given roots:

-5 and 8

Another way to solve a quadratic is by factoring an equation in standard form. The factoring techniques, or patterns, you learned will help you factor polynomials. Then you can use the Zero Product Property to solve equations.

**Zero Product Property**
For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal zero.

**Example 2: Solve Equations by Factoring**

a) $x^2 - 3x - 28 = 0$

b) $x^2 = 81$

c) $4x^2 = -3x$

d) $4x^2 - 17x = -4$

## 5.5 Completing the Square

When a quadratic equation contains a **perfect square trinomial set equal to a constant**, use the

**Example 1: Square Root Property**

a) $x^2 - 12x + 36 = 25$

b) $x^2 + 8x + 16 = 20$

The Square Root Property can only be used to solve quadratic equations when the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, use the following method:

______________________________________________

**Step 1:** ______________________________________ **Example:** ________________

**Step 2:** ______________________________________ ________________

**Step 3:** ______________________________________ ________________

**Therefore, the trinomial can be written as**

________________________________ ________________

**Example 2: Complete the Square**
Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

a) $x^2 - 14x + c$

c = _____ perfect square: ___________

b) $x^2 + 16x + c$

c = _____ perfect square: ___________

**Example 3: Solve an Equation by Completing the Square**

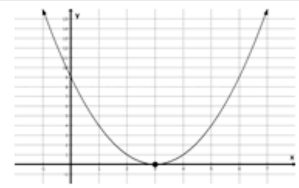
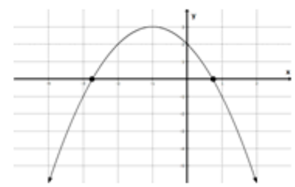
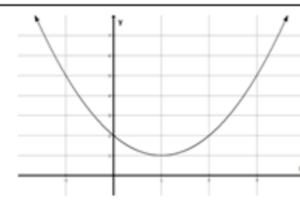
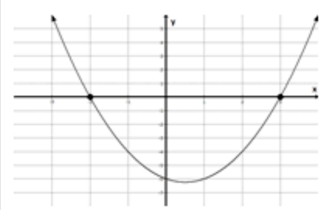
a) $x^2 + 8x - 84 = 0$

b) $x^2 + 14x - 1 = 0$

## 5.6 The Quadratic Formula and the Discriminant

**Quadratic Formula**
The solutions of a quadratic equation for the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula

**The Discriminant**
The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The following table summarizes the possible types of roots.

| Discriminant | Number of roots | Example | Graph |
|---|---|---|---|
| | | $y = x^2 - 6x + 9$<br>$b^2 - 4ac =$<br>$(-6)^2 - 4(1)(9) =$<br>$36 - 36 = \mathbf{0}$ |  |
| | | $y = -x^2 - 2x + 2$<br>$b^2 - 4ac =$<br>$(-2)^2 - 4(-1)(2) =$<br>$4 + 8 = \mathbf{12}$ |  |
| | | $y = x^2 - 2x + 2$<br>$b^2 - 4ac =$<br>$(-2)^2 - 4(1)(2) =$<br>$4 - 8 = \mathbf{-4}$ |  |
| | | $y = x^2 - x - 6$<br>$b^2 - 4ac =$<br>$(-1)^2 - 4(1)(-6) =$<br>$1 + 24 = \mathbf{25}$ |  |

**Example 1: The Quadratic Formula, Roots, and the Discriminant**
Complete parts 1-3 for each quadratic equation.

**1**. Find the value of the discriminant.
**2.** Describe the number and type of roots.
**3.** Find the exact solutions by using the Quadratic Formula.

a) $x^2 - 16x + 64 = 0$

**1.** discriminant: ___________ **2.** #/type of roots: ___________ **3.** exact solutions: _______________________

b) $x^2 - 10x - 50 = 0$

**1.** discriminant: ___________ **2.** #/type of roots: ___________ **3.** exact solutions: ______________________

You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

| Method | Can Be Used | When To Use |
|---|---|---|
| **Graphing** (5.1) | | |
| **Factoring** (5.3) | | |
| **Square Root Property** (5.5) | | |
| **Completing the Square** (5.5) | | |
| **Quadratic Formula** (5.6) | | |

**Example 2: Pick a Method to Solve a Quadratic Equation**
Solve each equation by using the method of your choice. Find exact solutions.

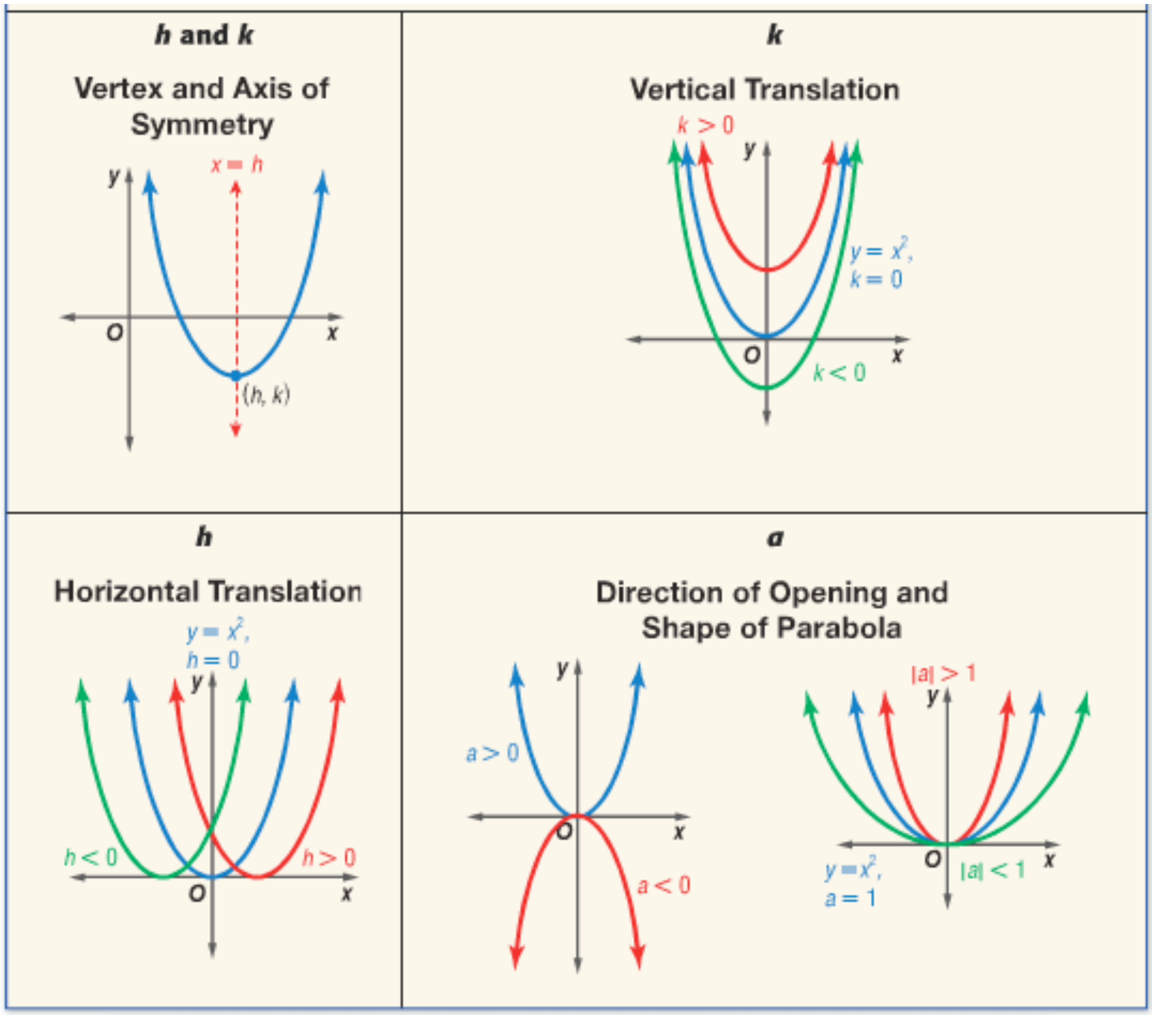
a) $3x^2 + 8x = 3$ Method Used: __________________________

b) $4x^2 - 12x + 7 = 0$ Method Used: __________________________

c) $4x^2 - 9 = 0$ Method Used: __________________________

**5.7 Analyzing Graphs of Quadratic Functions (Day One)**

**Vertex Form of a Quadratic Function**




**Example 1**: **Graph a Quadratic Equation in Vertex Form**
Identify the vertex, axis of symmetry, and direction of opening for each quadratic. Then graph.

a) $y = (x-3)^2 - 2$ **vertex:** __________ **axis of symmetry:** __________

**direction of opening:** __________

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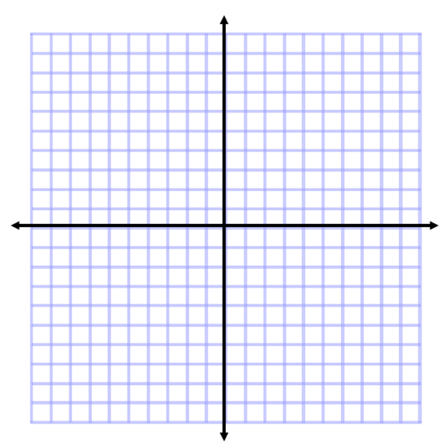

## 5.7 Analyzing Graphs of Quadratic Functions (Day One)

b) $y = 3(x+3)^2$ **vertex:** ___________ **axis of symmetry:** ___________

**direction of opening:** ___________

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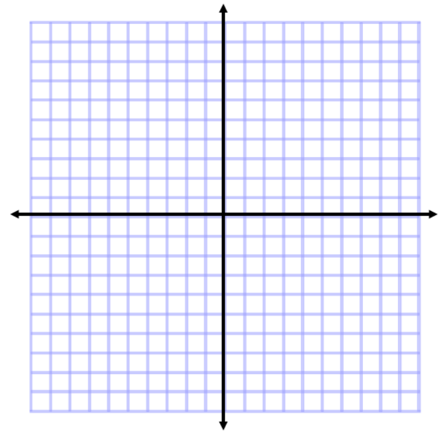
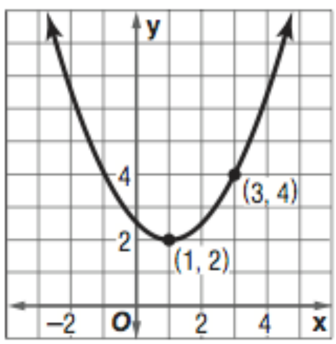


**<u>Example 2</u>: Write an Equation Given a Graph**

a)




b) Write an equation for the parabola whose vertex is at (2, 3) and passes through (-2, 1).

## 5.7 Analyzing Graphs of Quadratic Functions (Day Two)

Given a function of the form $y = ax^2 + bx + c$ , you can **complete the square** to write the function in vertex form.

**Example 1: Write Quadratic Equations in Vertex Form (when $a = 1$)**
Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph.

a) $y = x^2 + 4x + 6$

**vertex:** __________ **axis of symmetry:** __________

**direction of opening:** __________

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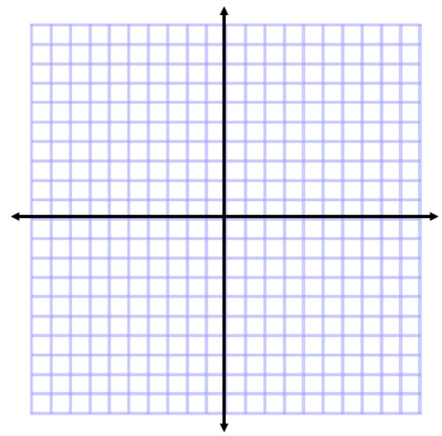

b) $y = x^2 + 2x + 4$

**vertex:** __________ **axis of symmetry:** __________

**direction of opening:** __________

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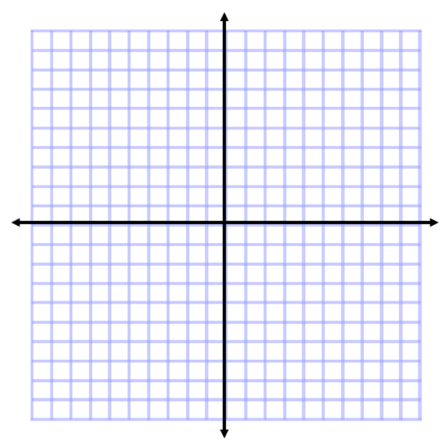

## 5.7 Analyzing Graphs of Quadratic Functions (Day Two)

**Example 2: Write Quadratic Equations in Vertex Form (when $a \neq 1$)**
Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph.

a) $y = -2x^2 - 4x + 2$

**vertex:** __________ **axis of symmetry:** __________

**direction of opening:** __________

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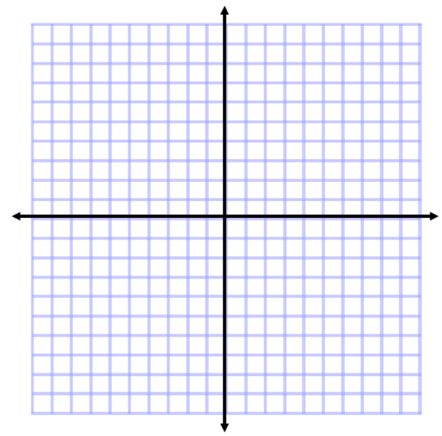

b) $y = 2x^2 + 12x + 17$

**vertex:** __________ **axis of symmetry:** __________

**direction of opening:** __________

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