



Chapter \_\_\_\_\_ :

Name \_\_\_\_\_

Period \_\_\_\_\_

#	Date	Assignment	Page	Score

Date	Test/Project	Score

## 9.1 Exponential Functions

### Exponential Functions

In an exponential function, the base is a constant and the exponent is a variable.

Domain:

Range:

Horizontal Asymptote:

### Example 1: Graph an Exponential Function

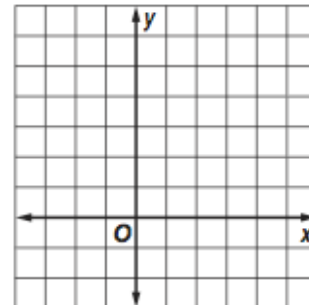
Sketch the graph of each function. Then state the function's domain and range.

a)  $y = 2(4)^x$

domain:

range:

h.a.:

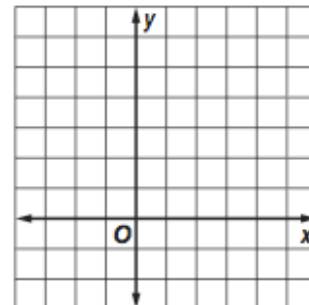



b)  $y = -2\left(\frac{1}{3}\right)^x$

domain:

range:

h.a.:

### Example 2: Identify Exponential Growth and Decay

Determine whether each function represents exponential growth or decay.

a)  $y = (0.5)^x$

b)  $y = 0.3(5)^x$

c)  $y = 2^{-x}$

### Example 3: Write an Exponential Function (in the form $y = ab^x$ )

Write an exponential function for the graph that passes through the given points.

a) (0, 3) and (-1, 6)

b) (0, -18) and (-2, -2)

## 9.1 Exponential Functions & 9.2 Logarithms and Logarithmic Functions

### Property of Equality for Exponential Functions

If  $b$  is a positive number other than 1, then...

### **Example 1: Solve Exponential Equations**

a)  $4^{2n-1} = 64$

b)  $5^{5x} = 125^{x+2}$

c)  $2^{n+4} = \frac{1}{32}$

### **Logarithm with Base $b$**

Suppose  $b > 0$  and  $b \neq 1$ . For  $x > 0$ , there is a number  $y$  such that...

### **Example 2: Logarithmic/Exponential Forms**

Write each equation in exponential or logarithmic form.

a)  $5^4 = 625$

b)  $\log_3 6 = \frac{1}{2}$

c)  $7^{-2} = \frac{1}{49}$

d)  $\log_3 81 = 4$

You can use the definition of logarithm to find the value of a logarithmic expression.

### **Example 3: Evaluate Logarithmic Expressions**

a)  $\log_3 243$

b)  $\log_4 256$

c)  $\log_2 \left( \frac{1}{8} \right)$

d)  $\log_6 216$

## 9.2 Logarithms and Logarithmic Functions

A **logarithmic equation** is an equation that contains one or more logarithms. You can use the definition of a logarithm to help you solve logarithmic equations.

### Example 1: Solve a Logarithmic Equation

a)  $\log_9 x = \frac{3}{2}$

b)  $\log_{16} x = \frac{5}{2}$

c)  $\log_b 9 = 2$

Use the following property to solve logarithmic equations that have logarithms with the same base on each side.

#### **Property of Equality for Logarithmic Functions**

If  $b$  is a positive number other than 1, then...

### Example 2: Solve Equations with Logarithms on Each Side

\*Extraneous solutions may exist within these problems, so ALWAYS CHECK YOUR SOLUTIONS!

a)  $\log_6(2x - 3) = \log_6(x + 2)$

b)  $\log_4 x^2 = \log_4(4x - 3)$

c)  $\log_3(x^2 - 15) = \log_3 2x$

d)  $\log_{14}(x^2 - 30) = \log_{14} x$

### 9.3 Properties of Logarithms (Day One)

#### Properties of Logarithms

**Product Property**

\*The logarithm of a product of numbers is the sum of the logarithms of the numbers.

**Quotient Property**

\*The logarithm of a quotient of numbers is the difference of the logarithms of the numbers.

**Power Property**

\*The logarithm of a power of a number is the exponent times the logarithm of the number.

**Example 1: Expanding Logarithmic Expressions**

Use the properties of logarithms to expand each expression in terms of  $\log x$ ,  $\log y$ ,  $\log z$ .

a)  $\log_2(6x)$

b)  $\log_5(x^3y^6)$

c)  $\log_3\left(\frac{xy}{\sqrt[3]{z}}\right)$

**Example 2: Combining Logarithmic Expressions**

Use the properties of logarithms to write each expression as a single logarithm.

a)  $\log_3 5 + 5 \log_3 2$

b)  $\log_2 x + \log_2 y - 2 \log_2 z$

c)  $4 \log_5 x - 2 \log_5(x - 1) + \frac{1}{3} \log_5(x^2 + 1)$

**Example 3: Using the Laws of Logarithms to Evaluate Expressions**

Write the following as a single logarithm and give its numerical value.

a)  $\log_4 2 + \log_4 32$

b)  $\log_2 80 - \log_2 5$

c)  $\log_2 6 - \log_2 15 + \log_2 20$

**Example 4: Rewriting Expressions**

Let  $x = \log_3 2$  and  $y = \log_3 7$ . Write each expression in terms of  $x$  and  $y$ .

a)  $\log_3 16$

b)  $\log_3 \frac{1}{49}$

c)  $\log_3 98$

### 9.3 Properties of Logarithms (Day Two)

In addition to the Product, Quotient, and Power Properties of Logarithms, there are other properties such as the four listed below that can be helpful.

$$\log_b 1 = \underline{\hspace{2cm}} \quad \log_b b = \underline{\hspace{2cm}} \quad \log_b b^x = \underline{\hspace{2cm}} \quad b^{\log_b x} = \underline{\hspace{2cm}}$$

You can use these properties, along with the properties of logarithms to solve equations involving logarithms.

**Example 1: Solve Equations with Logarithms on Both Sides**

a)  $2\log_7 x = \log_7 27 + \log_7 3$     b)  $\log_3 42 - \log_3 x = \log_3 7$     c)  $\log_2(3x) + \log_2 5 = \log_2 30$

**Example 2: Solve Equations with Logarithms on One Side**

a)  $\log_6 x + \log_6(x + 5) = 2$     b)  $\log_8 x + \log_8(x - 12) = 2$     c)  $\log_{10} x + \log_{10}(a + 21) = 2$

**Example 3: Applications**

Use the formula  $L = 10\log_{10} R$ , where L is the loudness of a sound and R is the sound's relative intensity, to find out how much louder 10 alarm clocks would be than one alarm clock.

a) Rewrite the formula to solve for the sound's relative intensity.

b) Find the sound's relative intensity if the sound of one alarm clock is 80 decibels.

## 9.4 Common Logarithms

You have seen that the base 10 logarithm function,  $y = \log_{10} x$ , is used in many applications. Base 10 logarithms are called **common logarithms**. Common logarithms are usually written without the subscript 10.

If both sides of an exponential equation cannot easily be written as powers of the same base, you can solve by taking the logarithm of each side.

### **Example 1: Solve Exponential Equations Using Logarithms**

Solve each equation. Round to four decimal places.

a)  $9^x = 45$

b)  $3 \cdot 1^{x-1} = 9.42$

c)  $11^{x^2} = 25.4$

d)  $7^{x-2} = 5^x$

e)  $2^{x+1} = 5^{2x-1}$

### **Example 2: Change of Base Formula**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

For all positive numbers,  $a$ ,  $b$ , and  $n$ , where  $a \neq 1$  and  $b \neq 1$ ,

a)  $\log_7 5$

b)  $\log_3 42$



## 9.5 Base e and Natural Logarithms

An exponential function with base e is called a **natural base exponential function**.

### **Example 1: Write Equivalent Expressions**

Write an equivalent exponential or logarithmic equation.

a)  $e^x = 6$

b)  $\ln x \approx 0.5352$

c)  $e^x = 4$

d)  $\ln 1 = 0$

Equations involving base e are easier to solve using natural logarithms than using common logarithms. All of the properties that you have learned apply to natural logarithms as well.

### **Example 2: Solve Base e Equations**

Solve each equation. Round to the nearest ten-thousandth.

a)  $2e^x - 5 = 1$

b)  $3 + e^{-2x} = 8$

c)  $3e^x + 2 = 4$

d)  $4e^{-x} - 9 = -2$

### **Example 3: Solve Natural Logarithmic Equations**

Solve each equation. Round to the nearest ten-thousandth.

a)  $\ln 3x = 7$

b)  $\ln x^2 = 9$

c)  $2 \ln 3x + 1 = 5$

## 9.6 Exponential Growth and Decay

Compound Interest	Exponential Growth	Exponential Decay
	1)	1)
Continuously Compounded Interest	2)	2)

### **Example 1: Calculating Compound Interest & Continuously Compound Interest**

A sum of \$1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, monthly, daily, and continuously.

### **Example 2: Exponential Decay**

- a) A store is offering a clearance sale on a certain type of digital camera. The original price for the camera was \$198. The price decreases 10% each week until all of the cameras are sold. How many weeks will it take for the price of the cameras to drop below half of the original price?
- b) A specimen that originally contained 150 milligrams of Carbon-14 now contains 130 milligrams. How old is the fossil? **The value of  $k$  for Carbon-14 is 0.00012 (pg. 545).**

### **Example 3: Exponential Growth**

- a) Home values in Millersport increase about 4% per year. Mr. Thomas purchased his home eight years ago for \$122,000. What is the value of his home now?
- b) The population of rabbits in an area is modeled by the growth equation  $P(t) = 8e^{0.26t}$ , where  $P$  is in thousands and  $t$  is in years. How long will it take for the population to reach 25,000?