8.1 Multiplying and Dividing Rational Expressions

To write a fraction in simplest form, you divide both the numerator and denominator by their greatest common factor (GCF). To simplify a rational expression, you use similar techniques.

Example 1: Simplify a Rational Expression

a) $\dfrac{45mn^3}{20n^7}$

b) $\dfrac{36c^3d^2}{54cd^4}$

c) $\dfrac{x^2+6x+9}{x+3}$

d) $\dfrac{9y^2-6y^3}{2y^2+5y-12}$

* **REMEMBER**: Factoring out a -1 in the numerator or denominator can help simplify rational expressions.

KEY CONCEPT *Rational Expressions*

Multiplying Rational Expressions

Words To multiply two rational expressions, multiply the numerators and the denominators.

Symbols For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.

Dividing Rational Expressions

Words To divide two rational expressions, multiply by the reciprocal of the divisor.

Symbols For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$ and $d \neq 0$.

Example 2: Multiply and Divide Rational Expressions

a) $\dfrac{8t^2s}{5r^2} \cdot \dfrac{15sr}{12t^3s^2}$

b) $\dfrac{9m^2n^3}{16ab^4} \cdot \dfrac{8a^2b}{27m^5n}$

c) $\dfrac{18ab^2}{25x^2y^3} \div \dfrac{9b}{10xy}$

d) $\dfrac{14pq^2}{15w^7z^3} \div \dfrac{21p^3q}{35w^3z^8}$

8.1 Multiplying and Dividing Rational Expressions

Sometimes you must factor the numerator and/or denominator first before you can simplify a product or a quotient of rational expressions.

Example 3: Polynomials in the Numerator and Denominator

a) $\frac{12p^2+6p-6}{4(p+1)^2} \cdot \frac{2p+10}{6p-3}$

b) $\frac{x^2+6x+9}{x^2+7x+6} \div \frac{4x+12}{3x+3}$

c) $\frac{x^2-5x-24}{6x+2x^2} \cdot \frac{5x^2}{8-x}$

d) $\frac{2m-1}{m^2-3m-10} \div \frac{4m^2-1}{4m+8}$

Complex fractions are no different from dividing rational expressions. To simplify a complex fraction,

____, and use the rules for division.

Example 4: Simplify a Complex Fraction

a) $\frac{\frac{(x+3)^2}{x^2-16}}{\frac{x+3}{x+4}}$

b) $\frac{\frac{y-7}{y-3}}{\frac{y^2-49}{y^2+4y-21}}$

8.2 Adding and Subtracting Rational Expressions

Least Common Multiple (LCM) of Polynomials
To find the LCM of two or more numbers or polynomials, ____.

The LCM contains each factor the ____________________ number of times it appears as a factor.

LCM of 6 and 4	**LCM of $a^2 - 6a + 9$ and $a^2 + a - 12$**
$6 = 2 \cdot 3$	$a^2 - 6a + 9 = (a - 3)^2$
$4 = 2^2$	$a^2 + a - 12 = (a - 3)(a + 4)$
$\text{LCM} = 2^2 \cdot 3$ or 12	$\text{LCM} = (a - 3)^2(a + 4)$

Example 1: LCM of Monomials and Polynomials
Find the LCM of each set of polynomials.

a) $12y^2, 6x^2$

LCM: ____________________________

b) $16ab^3, 5a^2b^2, 20ac$

LCM: ____________________________

c) $q^2 - 4q + 4, q^3 - 3q^2 + 2q$

LCM: ____________________________

d) $2k^3 - 5k^2 - 12k, k^3 - 8k^2 + 16k$

LCM: ____________________________

As with fractions, to add or subtract rational expression, you must have common denominators. You can **use the least common multiple of the denominators to find the least common denominator** for two rational expressions.

Example 2: Monomial Denominators

a) $\dfrac{1}{8m^2n} + \dfrac{2}{mn^2}$

b) $\dfrac{7a}{15b^2} - \dfrac{b}{18ab}$

c) $\dfrac{2}{x^2y} - \dfrac{x}{y}$

d) $\dfrac{6c}{7b^2} + \dfrac{2d}{3ab}$

8.2 Adding and Subtracting Rational Expressions

Example 3: Polynomial Denominators

a) $\frac{6}{d^2+4d+4}+\frac{5}{d+2}$

b) $\frac{x+6}{6x-18}+\frac{x-6}{2x-6}$

c) $\frac{5}{2x-12}+\frac{20}{x^2-4x-12}$

d) $\frac{x-1}{3x^2+8x+5}-\frac{x-1}{12x+20}$

Example 4: Simplify Complex Fractions

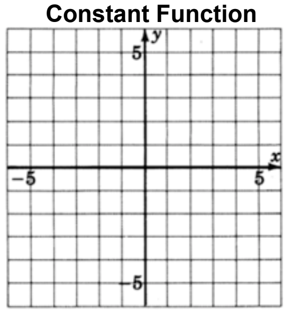
a) $\frac{x+\frac{x}{3}}{x-\frac{x}{6}}$

b) $\frac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{y}-\frac{1}{x}}$

8.5 Classes of Functions

In this book, you have studied several types of graphs representing special functions. The following is a summary of these graphs.

Constant Function



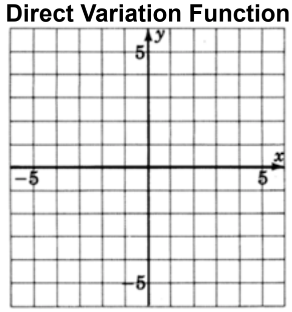
Gen. Eq. ______________

Important characteristics:

____

____

Direct Variation Function



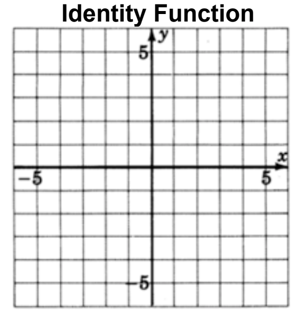
Gen. Eq. ______________

Important characteristics:

____

____

Identity Function



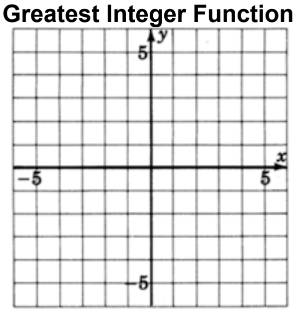
Gen. Eq. ______________

Important characteristics:

____

____

Greatest Integer Function



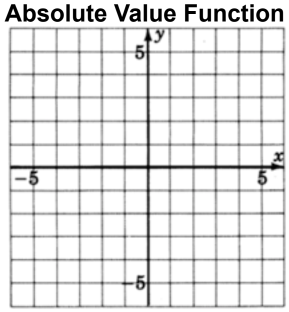
Gen. Eq. ______________

Important characteristics:

____

____

Absolute Value Function



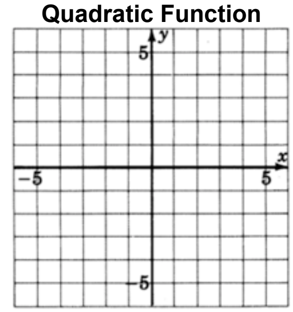
Gen. Eq. ______________

Important characteristics:

____

____

Quadratic Function



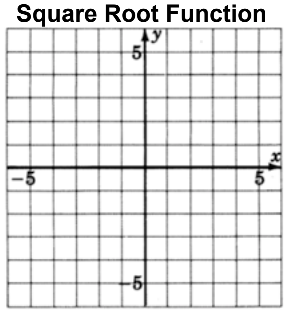
Gen. Eq. ______________

Important characteristics:

____

____

Square Root Function



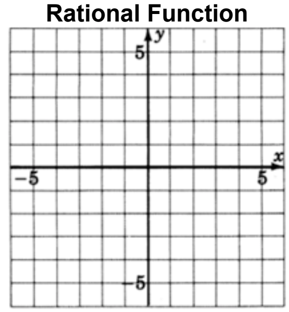
Gen. Eq. ______________

Important characteristics:

____

____

Rational Function



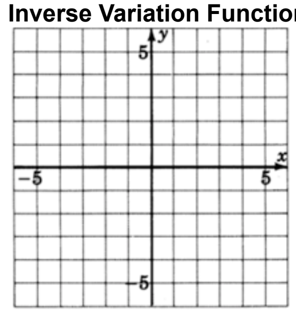
Gen. Eq. ______________

Important characteristics:

____

____

Inverse Variation Function



Gen. Eq. ______________

Important characteristics:

____

____

8.5 Classes of Functions

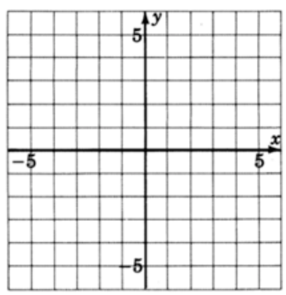
If you can identify an equation as a type of function, you can determine the shape of the graph, which can help you graph the function.

You can use the following transformations to help you graph each function using what you know about the parent graph.

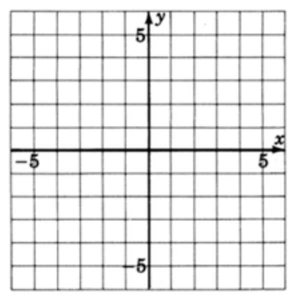
Function	Graph of the New Function	What Happens?
y = f(x - c)	Shift graph of y = f(x) right by c units	Add c to x-values
y = f(x + c)	Shift graph of y = f(x) left by c units	Subtract c from x-values
y = f(x) + c	Shift graph of y = f(x) upward by c units	Add c to y-values
y = f(x) - c	Shift graph of y = f(x) downward by c units	Subtract c from y-values
y = cf(x)	Stretch graph of y = f(x) vertically by factor of c	Multiply y-values by c
y = (1/c)f(x)	Compress graph of y = f(x) vertically by factor of c	Multiply y-values by 1/c
y = f(cx)	Compress graph of y = f(x) horizontally by factor of c	Multiply x-values by 1/c
y = f(x/c)	Stretch graph of y = f(x) horizontally by factor of c	Multiply x-values by c
y = - f(x)	Reflect graph of y = f(x) about x-axis	Multiply y-values by -1
y = f(-x)	Reflect graph of y = f(x) about y-axis	Multiply x-values by -1

<u>Example 1</u>: Identify a Function Given its Equation
Identify each type of function represented by each equation. The graph the equation.

a) $y = -x^2 + 2$ ______________ function

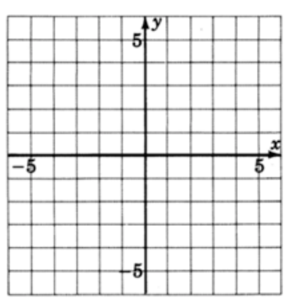


Parent Graph

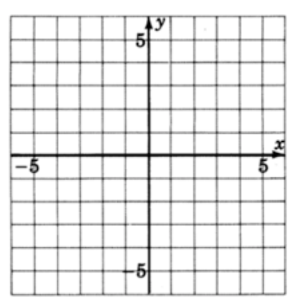


Transformed Graph

b) $y = |2x|$ ______________ function



Parent Graph



Transformed Graph

8.6 Solving Rational Equations (Day One)

Any equation that contains one or more rational expressions is called a ____________________________.

They are easier to solve if the fractions are **eliminated**. You can eliminate the fraction by multiplying each side of the equation by the ____.

Example 1: Solve a Rational Equation

a) $\frac{2}{d}+\frac{1}{4}=\frac{11}{12}$

b) $\frac{5}{6}+\frac{2}{x-6}=\frac{1}{2}$

c) $\frac{1}{x-1}+\frac{2}{x}=0$

d) $\frac{12}{v^2-16}-\frac{24}{v-4}=3$

Example 2: Eliminate a Possible Solution

* When solving a rational equation, **any possible solution that results in a zero in the denominator** must be excluded from your list of solutions.

a) $\frac{2}{r+1}-\frac{1}{r-1}=\frac{-2}{r^2-1}$

b) $\frac{7n}{3n+3}-\frac{5}{4n-4}=\frac{3n}{2n+2}$

8.6 Solving Rational Equations (Day Two)

Time needed to do a job and distance-speed-time problems frequently involve rational equations.

Example 1: Time Needed to a Job

FORMULA:

a) Breanne and Owen paint houses together. If Breanne can paint a particular house in 6 days and Owen can paint the same house in 5 days, how long would it take the two of them if they work together?

Equation: ______________________

b) Stan and Hilda can mow the lawn in 40 minutes if they work together. If Hilda does the work in half the time Stan does, how long does it take Stan to mow the lawn alone?

Equation: ______________________

Example 2: Distance-Speed-Time

FORMULA:

a) The speed of the current in a body of water is 1 mile per hour. Juan swims 2 miles against the current and 2 miles with the current in a total time of 2 2/3 hours. How fast can Juan swim in still water?

Equation: ______________________

b) Wendy took a trip from Davenport to Omaha, a distance of 300 miles. She traveled part of the way by bus, which arrived at the train station just in time for Wendy to complete her journey by train. The bus averaged 40 mph and train 60 mph. The entire trip took 5 1/2 hours. How long did Wendy spend on the train?

Equation: ______________________