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7.1 Operations on Functions

Let *f(x)* and *g(x)* be any two functions. You can add, subtract, multiply, and divide functions according to these rules.

Operation	Definition
Sum	
Difference	
Product	
Quotient	

Example 1: Performing Operations with Functions

Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find each function.

a) $(f + g)(x) =$

b) $(f - g)(x) =$

Given $f(x) = x^2 - 7x + 2$ and $g(x) = x + 4$, find each function.

a) $(f \cdot g)(x) =$

b) $\left(\frac{f}{g}\right)(x) =$

Example 2: Application

The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when *x* items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function *C(x)* for the total manufacturing and service cost per item.

7.2 Inverse Functions and Relations

- **inverse relation** - the set of ordered pairs obtained by ____________________ the coordinates of each ordered pair

Example 1: Find an Inverse Relation
Find the inverse of each relation.

a) {(-8, -3), (-8, -6), (-3, -6)} **inverse relation:** ____

b) {(1, 3), (6, 3), (6, 0), (1, 0)} **inverse relation:** ____

The way you find an inverse function is similar to finding an inverse relation.

Steps to Finding an Inverse Function

Step 1: ____

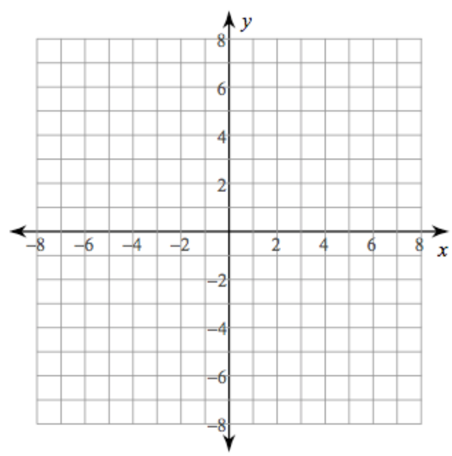
Step 2: ____

Step 3: ____

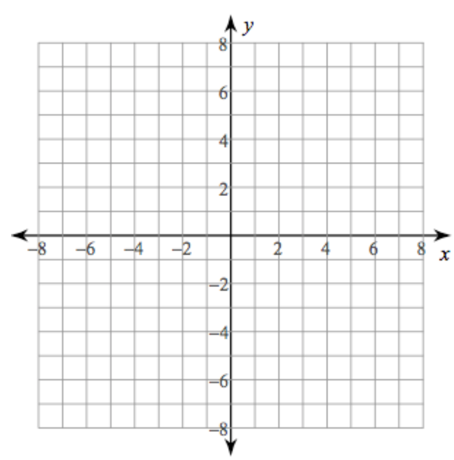
Step 4: ____

Example 2: Find and Graph an Inverse Function
Find the inverse of each function. Then graph the function and its inverse.

a) $f(x) = \dfrac{x-3}{5}$



b) $f(x) = -\dfrac{1}{2}x + 1$



7.2 Inverse Functions and Relations

You can determine whether two functions are inverses by finding both of their ________________________.

If both equal the **identity function** ______________, then the functions are inverse functions.

KEY CONCEPT ***Inverse Functions***

Words Two functions f and g are inverse functions if and only if both of their compositions are the identity function.

Symbols $[f \circ g](x) = x$ and $[g \circ f](x) = x$

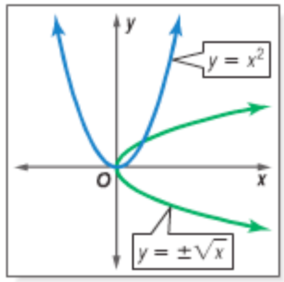
Example 3: Verify that Two Functions are Inverses
Determine whether each pair of functions are inverse functions.

a) $f(x) = 3x - 3$
$g(x) = \frac{1}{3}x + 4$

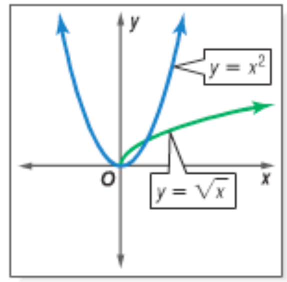
b) $f(x) = \frac{3}{4}x - 6$
$g(x) = \frac{4}{3}x + 8$

7.3 Square Root Functions and Inequalities

Square Root Functions
If a function contains a square root of a variable, it is called a ________________________________.



$y = \pm\sqrt{x}$ is not a function.



$y = \sqrt{x}$ is a function.

In order for a square root to be a real number, the radicand **cannot be** __________________________. When graphing a square root function, determine when the radicand would be negative and **exclude them from the domain**.

Examples: Graphing Square Root Functions
For each function, (a) state the starting point, (b) complete a t-chart with two other points, (c) state the domain & range, and then (d) graph.

1) $y = \sqrt{2x}$

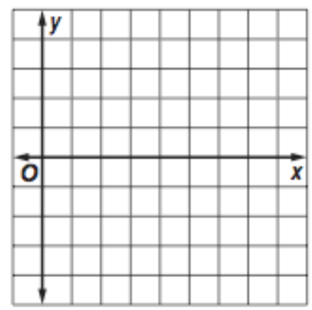
a)

b)

c) domain:

range:

d)



2) $y = 2\sqrt{x+3}$

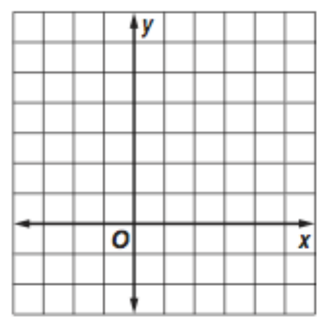
a)

b)

c) domain:

range:

d)



3) $y = \sqrt{x+4} - 2$

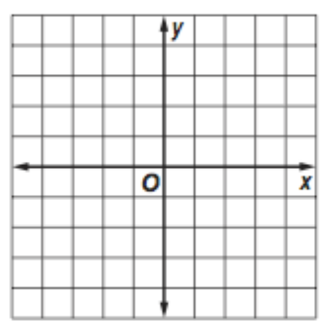
a)

b)

c) domain:

range:

d)



7.4 nth Roots

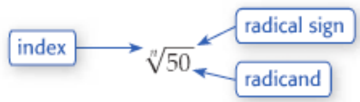
Simplifying Radicals
Finding the square of a number and squaring a number are inverse operations. The inverse of raising a number to the *n*th power is finding the ***n*th root** of a number.

KEY CONCEPT — ***Definition of nth Root***

Word For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.

Example Since $2^5 = 32$, 2 is a fifth root of 32.

The symbol $\sqrt[n]{}$ indicates an nth root.



The type of *n*th root depends on the radicand and the index.

Index	Positive Radicand	Negative Radicand	Zero Radicand
even			
odd			

Example 1: Find Roots

a) $\pm\sqrt{81y^6}$

b) $-\sqrt{(x-3)^{12}}$

c) $\sqrt[6]{729x^{30}y^{18}}$

d) $\sqrt{-25}$

e) $-\sqrt{(q^3+5)^4}$

f) $\sqrt[5]{243a^{10}b^{15}}$

g) $\pm\sqrt{16x^8}$

h) $\sqrt{-4}$

Example 2: Simplify Using Absolute Value
*When you find the *n*th root of an **even power** and the result is an **odd power**, you must take the **ABSOLUTE VALUE of the result** to ensure that the power is nonnegative.

a) $\sqrt{100x^{10}}$

b) $\sqrt{64(x+1)^{14}}$

c) $\sqrt[5]{243(x+2)^{15}}$

7.5 Operations with Radical Expressions (Day One)

Simplify Radicals

The properties you have used to simplify radical expressions involving square roots also hold true for expressions involving *n*th roots.

KEY CONCEPT *Properties of Radicals*

For any real numbers a and b and any integer $n > 1$, the following properties hold true.

Property	Words	Examples
Product Property	**1.** If n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and **2.** If n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.	$\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$, or 4, and $\sqrt[3]{3} \cdot \sqrt[3]{9} = \sqrt[3]{27}$, or 3
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined and $b \neq 0$.	$\frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27}$, or 3

Follow these steps to simplify a square root:

Step 1: ______________________

____

Step 2: ______________________

____

Step 3: ______________________

A radical expression is in simplified form when the following conditions are met:

1. ______________________
2. ______________________

   ____
3. ______________________
4. ______________________

Example 1: Simplify Radicals

a) $\sqrt{36r^5s^{10}}$

b) $\sqrt{25a^4b^9}$

Example 2: Multiply Radicals

a) $5\sqrt[4]{24x^3} \cdot 4\sqrt[4]{54x}$

b) $7\sqrt[3]{75a^4} \cdot 3\sqrt[3]{45a^2}$

7.5 Operations with Radical Expressions (Day One)

You add radicals in the same manner as adding monomials. That is, you can add only the like terms or like radicals. Two radical expressions are called **like radical expressions** if **BOTH the indices and the radicals are alike.**

Like:	$2\sqrt[4]{3a}$ and $5\sqrt[4]{3a}$	Radicands are $3a$; indices are 4.
Unlike:	$\sqrt{3}$ and $\sqrt[3]{3}$	Different indices
	$\sqrt[4]{5x}$ and $\sqrt[4]{5}$	Different radicands

Example 3: Add and Subtract Radicals

a) $3\sqrt{8}+5\sqrt{32}-4\sqrt{18}$

b) $5\sqrt{12}-2\sqrt{27}+6\sqrt{108}$

Example 4: Multiply Radicals Using FOIL

a) $(4\sqrt{2}+2\sqrt{6})(\sqrt{5}-3)$

b) $(3\sqrt{5}+4)(3\sqrt{5}-4)$

7.5 Operations with Radical Expressions (Day Two)

Simplify Radicals

The properties you have used to simplify radical expressions involving square roots also hold true for expressions involving *n*th roots.

KEY CONCEPT — *Properties of Radicals*

For any real numbers a and b and any integer $n > 1$, the following properties hold true.

Property	Words	Examples
Product Property	**1.** If n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and **2.** If n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.	$\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$, or 4, and $\sqrt[3]{3} \cdot \sqrt[3]{9} = \sqrt[3]{27}$, or 3
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined and $b \neq 0$.	$\frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27}$, or 3

REMEMBER!!! A radical expression is in simplified form when the following conditions are met.

1. ____

2. ____

3. ____

4. ____

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1: Simplify Expressions

a) $\sqrt{\dfrac{m^9}{n^7}}$ b) $\sqrt[3]{\dfrac{2}{9x}}$ c) $\sqrt[4]{\dfrac{3x}{2}}$

You can also use **conjugates** to rationalize denominators.

Example 2: Use a Conjugate to Rationalize a Denominator

a) $\dfrac{4+\sqrt{2}}{5-\sqrt{2}}$ b) $\dfrac{3-2\sqrt{5}}{6+\sqrt{5}}$

Name________________________
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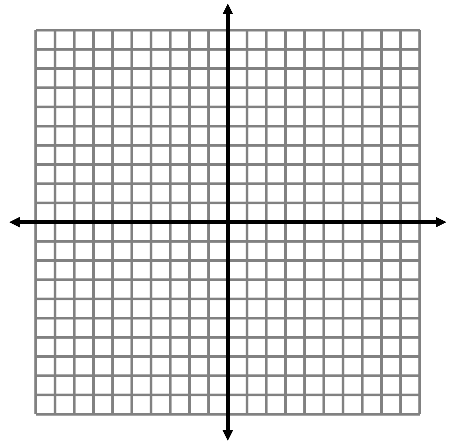
SQUARE ROOT FUNCTION INVESTIGATION

Graph $y = \sqrt{x}$ by completing the chart below.

x	y
9	
4	
1	
0	
-1	
-4	
-9	



Describe the shape of the graph?____All graphs of square root function have this the same shape. What is the domain of this function?______________________________________ What is the range of this function?____

The rest of this worksheet will lead you through an investigation of the graph of the square root function. The changes that occur to the graph include the coordinates of the "starting" point of the graph, whether the graph is the top or bottom half of a sideways parabola and the domain and range of the function.

As you complete this investigation, you should find many similarities to past investigations that have been done this year Enter the functions in the **y=** menu one at a time. Display y_1 before you enter y_2. Display y_1 and y_2 before you enter y_3. Display all three functions after you enter y_3. Any expression under the square root symbol must be in parentheses.

(Use the following window settings: $X\min = -11.75$ $X\max = 11.75$ $Xscl = 1$ $Y\min = -7.75$ $Y\max = 7.75$ $Yscl = 1$)

$y_1 = \sqrt{x}$

$y_2 = \sqrt{x} + 2$ Starting point:________ Domain: _______________________ Range: _______________________

$y_3 = \sqrt{x} - 4$ Starting point:________ Domain: _______________________ Range: _______________________

What effect does the letter ***k*** have on the graph of $y = \sqrt{x} + k$? ____

Clear out the three functions and enter the following three functions in the same manner as above.

$y_1 = \sqrt{x}$

$y_2 = \sqrt{x+5}$ Starting point:________ Domain: _______________________ Range: _______________________

$y_3 = \sqrt{x-3}$ Starting point:________ Domain: _______________________ Range: _______________________

What effect does the letter ***h*** have on the graph of $y = \sqrt{x-h}$?____

Clear out the three functions and enter the following three functions in the same manner as above.

$y_1 = \sqrt{x}$

$y_2 = \sqrt{x-1} + 4$ Starting point:________ Domain: _______________________ Range: _______________________

$y_3 = \sqrt{x+5} - 7$ Starting point:________ Domain: _______________________ Range: _______________________

What will be the starting point of $y = \sqrt{x-4} + 6$? ________ Graph it and see if you are correct.

In general, what will be the starting point of $y = \sqrt{x-h} + k$? ________________

Clear out the three functions and enter the following three functions in the same manner as above.

$y_1 = \sqrt{x}$

$y_2 = 3\sqrt{x}$ Starting point:________ Domain: ______________________ Range: ______________________

$y_3 = \frac{1}{2}\sqrt{x}$ Starting point:________ Domain: ______________________ Range: ______________________

What does the letter ***a*** do to the graph of $y = a\sqrt{x}$?____

Clear out the three functions and enter the following two functions in the same manner as above.

$y_1 = \sqrt{2x-6}$ Starting point:________ Domain: ______________________ Range: ______________________

$y_2 = \sqrt{2(x-3)}$ Starting point:________ Domain: ______________________ Range: ______________________

What do you notice about the graphs of these two functions?____
Why is that?____

Clear out the two functions.

How could $y = \sqrt{6x+12}$ be rewritten?______________________________________
Graph both functions. Are they the same? _____ What is the starting point?______________

What will be the starting point of $y = \sqrt{3x-15}$? __________ Graph it an see if you are correct.

What will be the starting point of $y = \sqrt{4x+10}$? __________ Graph it an see if you are correct.
(NOTE: The coefficient of the x, whether it is under the square root or not, will have a stretching or compression effect, depending if its absolute value is more than 1 or less than one.

Clear out any functions and enter the following two functions in the same manner as above. Answer the questions that follow.

$y_1 = \sqrt{x}$

$y_2 = -\sqrt{x}$ Starting point:________ Domain: ______________________ Range: ______________________

What effect does the $-$ have on the graph?____

Is $y = \sqrt{x}$ an increasing or decreasing function?________________________

Is $y = -\sqrt{x}$ an increasing or decreasing function?________________________

Describe the graph of each of the following. Give the starting point, the domain and the range for each function, and tell whether the function is increasing or decreasing. After describing each function, graph it on the graphing calculator and see if you are correct.

$y = \sqrt{5x+10} - 4$

$y = -3\sqrt{x-2} + 1$

$y = \sqrt{2x-7} - 4$

Will a square root graph ever open to the left instead of to the right?_______What would the equation of such a function look like?_______________________

When graphing a square root function by hand, plot its starting point and think about the domain and range of the function as well as whether the function is increasing or decreasing. Plug in a few more x values and plot the resulting ordered pairs. These ordered pairs should satisfy what you thought about the domain, the range and whether the function was increasing or decreasing.

7.6 Fractional Exponents

The Relationship Between Fractional Exponents and Radicals

Example 1: Radical and Exponential Forms
Write each expression in radical form, or write each radical using fractional exponents.

a) $11^{\frac{1}{7}}$ b) $6^{\frac{2}{5}}$ c) $(n^3)^{\frac{2}{5}}$

d) $\sqrt{47}$ e) $\sqrt[3]{3a^5b^2}$ f) $\sqrt[4]{162p^5}$

Example 2: Evaluate Expression with Fractional Exponents

a) $27^{-\frac{1}{3}}$ b) $64^{\frac{2}{3}}$ c) $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$ d) $\frac{64^{2/3}}{343^{2/3}}$

Example 3: Simplify Expressions with Fractional Exponents

a) $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ b) $(u^{-\frac{1}{3}})^{-\frac{4}{5}}$ c) $b^{-\frac{3}{5}}$ d) $\frac{q^{3/5}}{q^{2/5}}$

Example 4: Simplify Radical Expressions

a) $\frac{\sqrt[4]{32}}{\sqrt[3]{2}}$ b) $\sqrt[3]{16x^4}$ c) $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ d) $\frac{a}{\sqrt{3b}}$

7.7 Solving Radical Equations and Inequalities (Day One)

Equations with radicals that have variables in the radicands are called **radical equations**. To solve this type of equation, raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

It is **VERY IMPORTANT to check your solution**. Sometimes you will obtain a number that does not satisfy the original equation. Such a number is called an **extraneous solution**.

Example 1: Solve Radical Equations with One Radical

a) $5 = \sqrt{x-2} - 1$

b) $12 + \sqrt{2x-1} = 4$

c) $6 + \sqrt[3]{q-4} = 9$

Example 2: Solve Radical Equations with Two Radicals

a) $\sqrt{2d-5} - \sqrt{d-1} = 0$

b) $\sqrt{x+15} = 5 + \sqrt{x}$

c) $\sqrt{3x+1} = \sqrt{5x} - 1$

Example 3: Solve Radical Equations with Fractional Exponents

a) $(3n+2)^{\frac{1}{3}} + 1 = 0$

b) $(7v-2)^{\frac{1}{4}} + 12 = 7$

c) $(3g+1)^{\frac{1}{2}} - 6 = 4$