

Chapter _____ : _____

Name _____

Period _____

#	Date	Assignment	Page	Score

Date	Test/Project	Score

6.1 Properties of Exponents (With Negative Exponents)

Name of Property	Formula
1. _____	_____
2. _____	_____
3. _____	_____
4. _____	_____
5. _____	_____
6. _____	_____
7. _____	_____

Simplifying Expressions

A monomial expression is in simplified form when:

- _____
- _____
- _____

Example 1: Simplifying Expressions Using Several Properties

Simplify. Assume that no variable equals 0.

a) $(4d^2t^5v^{-4})(-5dt^{-3}v^{-1})$

b) $(-2b^{-2}c^3)^3$

c) $\frac{(-m^4)^3}{(2m^2)^{-2}}$

d) $\left(\frac{2x^3y^2}{-x^2y^5}\right)^{-2}$

Example 2: Scientific Notation *A number is in scientific notation when it is in _____.

- The density of an object is equal to its mass divided by its volume. A dumbbell has a mass of 9×10^3 grams and a volume of 1.2×10^3 . What is the density of the dumbbell?
- When light passes through water, its velocity is reduced by 25%. If the speed of light in a vacuum is 1.86×10^5 miles per second, at what velocity does it travel through water? Write your answer in scientific notation.

6.3 Dividing Polynomials (Day One)

In section 6.1, you learned to divide monomials. You can divide a polynomial by a monomial by using those same skills.

Example 1: Divide a Polynomial by a Monomial

a) $\frac{9x^2y^3 - 15xy^2 + 12xy^3}{3xy^2}$

b) $\frac{16a^5b^3 + 12a^3b^4 - 20ab^5}{4ab^3}$

c) $(20c^4d^2f - 16cf + 4cdf)(4cdf)^{-1}$

d) $(18x^2y + 27x^3y^2z)(3xy)^{-2}$

Example 2: Long Division (Quotient with No Remainder)

a) $(x^2 + 7x - 30) \div (x - 3)$

b) $(x^3 - 3x^2 + x - 3) \div (x^2 + 1)$

Example 3: Long Division (Quotient with Remainder)

a) $(x^2 - 3x - 7) \div (x + 2)$

b) $(2x^3 + 3x - 14) \div (x + 3)$

6.3 Dividing Polynomials (Day Two)

Synthetic Division

A simpler process for dividing a polynomial by a binomial. To use synthetic division, the divisor must be of the form...

LONG DIVISION VS SYNTHETIC DIVISION

Use **long division** to find:

$$(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$$

= _____

Use **synthetic division** to find:

$$(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$$

$$\begin{array}{r|rrrr} & & & & \\ \hline & & & & \\ & & & & \\ & & & & \end{array}$$

= _____

Example 1: Synthetic Division (Divisor with First Coefficient 1)

Use synthetic division to find each quotient.

a) $(2x^3 + 3x^2 - 4x + 15) \div (x + 3)$

$$\begin{array}{r|rrrr} & & & & \\ \hline & & & & \\ & & & & \\ & & & & \end{array}$$

= _____

b) $(3x^3 - 8x^2 + 11x - 14) \div (x - 2)$

$$\begin{array}{r|rrrr} & & & & \\ \hline & & & & \\ & & & & \\ & & & & \end{array}$$

= _____

Example 2: Synthetic Division (Divisor with First Coefficient Other than 1)

Use synthetic division to find each quotient.

a) $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$

$$\begin{array}{r|rrrrr} & & & & & \\ \hline & & & & & \\ & & & & & \\ & & & & & \end{array}$$

= _____

b) $(8x^5 - 2x^4 - 16x^2 + 4) \div (4x - 1)$

$$\begin{array}{r|rrrrr} & & & & & \\ \hline & & & & & \\ & & & & & \\ & & & & & \end{array}$$

= _____

6.4 Polynomials Functions (Day One)

Polynomial in One Variable

- The **degree** of a polynomial in one variable is the _____ of its variable.
- The **leading coefficient** is the _____.

Example 1: Find Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- a) $7x^6 - 4x^3 + \frac{1}{x}$ **Polynomial?** Yes OR No _____
- b) $\frac{1}{2}x^2 + 2x^3 - x^5$ **Polynomial?** Yes OR No _____
- c) $7z^3 - 4z^2 + z$ **Polynomial?** Yes OR No _____
- d) $6a^3 - 4a^2 + ab^2$ **Polynomial?** Yes OR No _____

Example 2: Function Values of Variables

Find $p(3)$ and $p(-1)$ for each function.

a) $p(x) = -x^3 + x^2 - x$

$p(3) =$

$p(-1) =$

b) $p(x) = x^4 - 3x^3 + 2x^2 - 5x + 1$

$p(3) =$

$p(-1) =$

If $p(x) = 2x^3 + 6x - 12$ and $q(x) = 5x^2 + 4$, find each value.

a) $p(a^3)$

b) $5[q(2a)]$

c) $3p(a) - q(a + 1)$

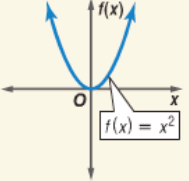
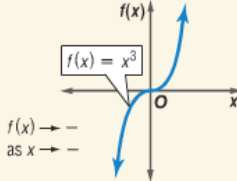
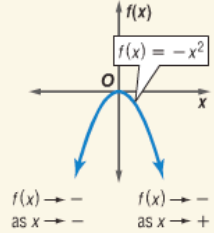
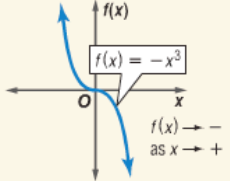
6.4 Polynomials Functions (Day One)

End Behavior

- is a description of what happens as x becomes large in the positive or negative direction
- is determined by the **degree** (highest exponent of polynomial) & the sign of the **leading coefficient**

CONCEPT SUMMARY

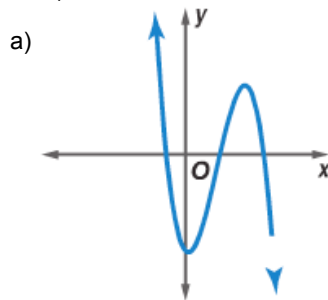
End Behavior of a Polynomial Function

<p>Degree: even Leading Coefficient: positive End Behavior:</p> <p>$f(x) \rightarrow +$ as $x \rightarrow -$ $f(x) \rightarrow +$ as $x \rightarrow +$</p>  <p>$f(x) = x^2$</p> <p>$y \rightarrow \infty$ as $x \rightarrow -\infty$ $y \rightarrow \infty$ as $x \rightarrow \infty$</p>	<p>Degree: odd Leading Coefficient: positive End Behavior:</p> <p>$f(x) \rightarrow -$ as $x \rightarrow -$ $f(x) \rightarrow +$ as $x \rightarrow +$</p>  <p>$f(x) = x^3$</p> <p>$y \rightarrow -\infty$ as $x \rightarrow -\infty$ $y \rightarrow \infty$ as $x \rightarrow \infty$</p>	<p>Degree: even Leading Coefficient: negative End Behavior:</p> <p>$f(x) \rightarrow -$ as $x \rightarrow -$ $f(x) \rightarrow -$ as $x \rightarrow +$</p>  <p>$f(x) = -x^2$</p> <p>$y \rightarrow -\infty$ as $x \rightarrow -\infty$ $y \rightarrow -\infty$ as $x \rightarrow \infty$</p>	<p>Degree: odd Leading Coefficient: negative End Behavior:</p> <p>$f(x) \rightarrow +$ as $x \rightarrow -$ $f(x) \rightarrow -$ as $x \rightarrow +$</p>  <p>$f(x) = -x^3$</p> <p>$y \rightarrow \infty$ as $x \rightarrow -\infty$ $y \rightarrow -\infty$ as $x \rightarrow \infty$</p>
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Example 3: Graphs of Polynomial Functions

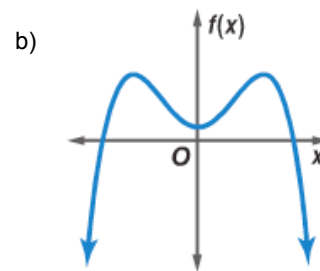
For each graph,

- 1) describe the end behavior,
- 2) determine whether it represents an odd-degree or an even-degree polynomial function, and
- 3) state the number of real zeros



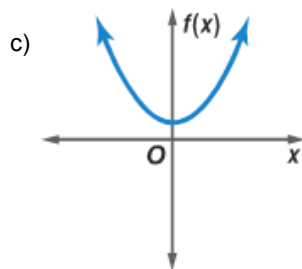
end behavior: _____

degree? _____ # of real zeros: _____



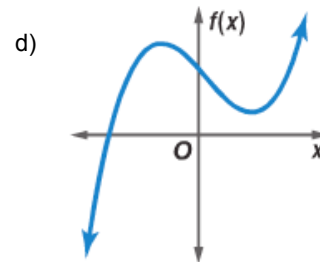
end behavior: _____

degree? _____ # of real zeros: _____



end behavior: _____

degree? _____ # of real zeros: _____



end behavior: _____

degree? _____ # of real zeros: _____

6.4 Polynomials Functions (Day Two)

In order to sketch the graph of a polynomial function, you need to:

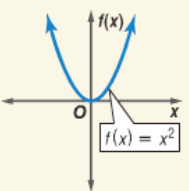
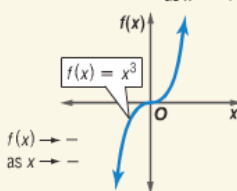
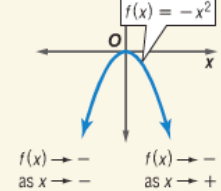
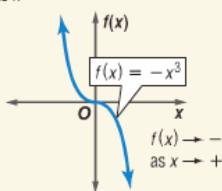
- (1) factor the polynomial to find all its **real zeros**; these are the x-intercepts of the graph
- (2) determine the **end behavior** of the polynomial
- (3) identify the **multiplicity** of each zero to determine the shape of the graph near each zero

End Behavior

- is a description of what happens as x becomes large in the positive or negative direction
- is determined by the **degree** (highest exponent of polynomial) & the sign of the **leading coefficient**

CONCEPT SUMMARY

End Behavior of a Polynomial Function

<p>Degree: even Leading Coefficient: positive End Behavior:</p> <p>$f(x) \rightarrow +$ as $x \rightarrow -$ $f(x) \rightarrow +$ as $x \rightarrow +$</p>  <p>$f(x) = x^2$</p> <p>$y \rightarrow \infty$ as $x \rightarrow -\infty$ $y \rightarrow \infty$ as $x \rightarrow \infty$</p>	<p>Degree: odd Leading Coefficient: positive End Behavior:</p> <p>$f(x) \rightarrow -$ as $x \rightarrow -$ $f(x) \rightarrow +$ as $x \rightarrow +$</p>  <p>$f(x) = x^3$</p> <p>$y \rightarrow -\infty$ as $x \rightarrow -\infty$ $y \rightarrow \infty$ as $x \rightarrow \infty$</p>	<p>Degree: even Leading Coefficient: negative End Behavior:</p> <p>$f(x) \rightarrow -$ as $x \rightarrow -$ $f(x) \rightarrow -$ as $x \rightarrow +$</p>  <p>$f(x) = -x^2$</p> <p>$y \rightarrow -\infty$ as $x \rightarrow -\infty$ $y \rightarrow -\infty$ as $x \rightarrow \infty$</p>	<p>Degree: odd Leading Coefficient: negative End Behavior:</p> <p>$f(x) \rightarrow +$ as $x \rightarrow -$ $f(x) \rightarrow -$ as $x \rightarrow +$</p>  <p>$f(x) = -x^3$</p> <p>$y \rightarrow \infty$ as $x \rightarrow -\infty$ $y \rightarrow -\infty$ as $x \rightarrow \infty$</p>
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Multiplicity

Given $P(x) = A(x - c)^m$, where c is a zero (x-intercept) of the polynomial and m is the multiplicity:

if m is odd, $m \geq 1$
if m is even, $m > 1$

then the graph **crosses the x-axis** through that given zero
then the graph **touches the x-axis** (does not cross) at that given zero

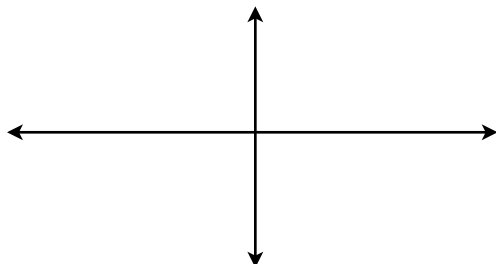
Example: Sketching Graphs of Polynomial Functions

a) $f(x) = -x^4 - x^3 + 6x^2$

Zeros:

End Behavior:

Multiplicity:

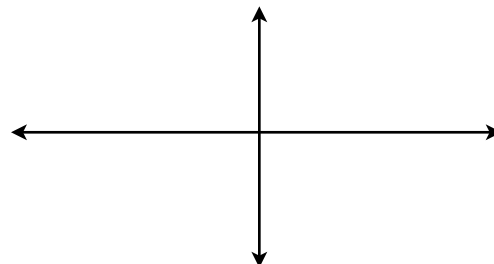


b) $f(x) = x^5 - 9x^3$

Zeros:

End Behavior:

Multiplicity:



Graphing Polynomials Worksheet

Graphing Polynomials

For each of the following polynomial functions, write the degree and find the given value.

1. $f(x) = x^3 - 3x^2 + 2x - 1; f(2)$

2. $g(x) = 3x - x^2 + x^4 - 2x^3; g(1)$

3. $h(x) = x^5 - x^3 + 1; h(3)$

For each of the following polynomial functions:

- find the zeros of the polynomial
- find the degree of the polynomial
- graph the polynomial. (HINT: Factor when necessary.)

4. $f(x) = x(x-2)(x+3)$

5. $g(x) = (x-1)(x-3)(x+4)$

6. $h(x) = (x-1)(x-3)^2$

7. $f(x) = x^2(x-4)$

8. $g(x) = -x(x+1)(x-3)$

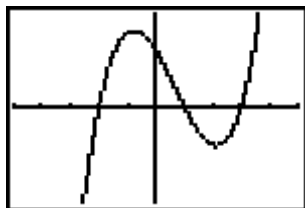
9. $h(x) = x^3 - 2x^2 - 8x$

10. $f(x) = x^4 - 4x^3 + 4x^2$

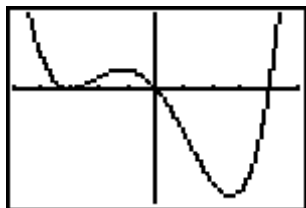
11. $g(x) = -2x^4 + 8x^2$

Write a possible polynomial function, in factored form, for each of the following graphs.

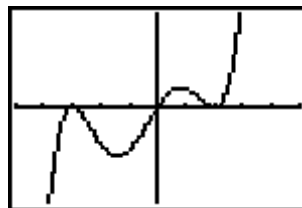
12.



13.



14.



15.



6.6 Solving Polynomial Equations

Factoring Techniques

The table below summarizes the most common factoring techniques used with polynomials.

CONCEPT SUMMARY		Factoring Techniques
Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$a^3b^2 + 2a^2b - 4ab^2 = ab(a^2b + 2a - 4b)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes Difference of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Whenever you factor a polynomial, **ALWAYS** go through the following steps:

Step 1: _____

Step 2: _____

Example 1: Factor Polynomials

Factor each polynomial completely.

a) $8yz - 6z - 12y + 9$

b) $y^2 - 5y + 4$

c) $z^3 + 125$

d) $t^3 - 8$

e) $3ax - 15a + x - 5$

f) $2b^2 + 13b - 7$

In Chapter 5, you learned to solve quadratic equations by factoring and using the Zero Product Property. You can extend these techniques to solve higher-degree polynomial equations.

Example 2: Solve Polynomial Equations

a) $x^4 - 29x^2 + 100 = 0$

b) $x^3 + 8 = 0$

c) $x^3 - 216 = 0$

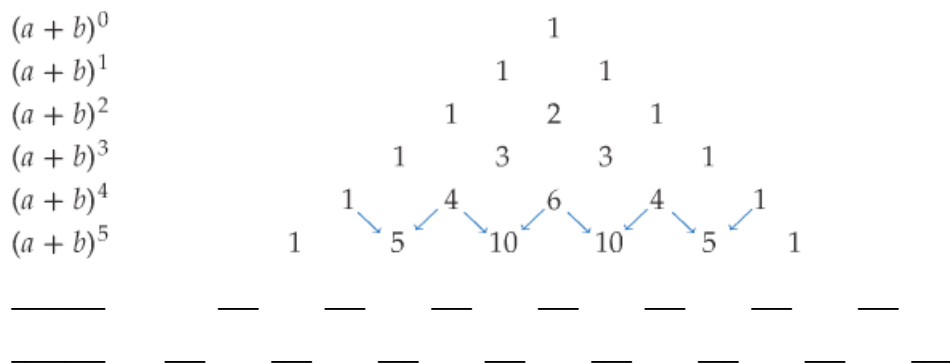
d) $x^4 - 6x^2 - 27 = 0$

11.7 The Binomial Theorem

Pascal's Triangle

A triangular array of numbers such that the $(n + 1)^{\text{th}}$ row is the coefficient of the terms of the expansion $(x + y)^n$ for $n = 0, 1, 2, \dots$

Notice that **each row begins and end with 1**, and **each coefficient is the sum of the two coefficients above it** in the previous row.



Example 1: Use Pascal's Triangle

Expand each power.

a) $(w + z)^5$

b) $(t - s)^6$

c) $(x + 2y)^5$

d) $(5x - 2y)^4$