Chapter _____: ____

Name ____ Period __________

#	Date	Section & Topic/Activity	Page

Chapter _____: ____

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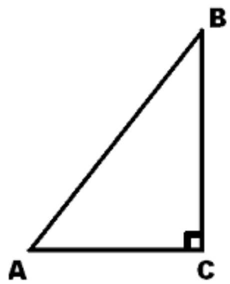
#	Date	Assignment	Page	Score

Date	Test/Project	Score

13.1 Right Triangle Trigonometry

<u>Trigonometry</u> is the study of the relationships among the angles and sides of a right triangle.



Using the sides of the right triangle, you can define **six trigonometric functions**.

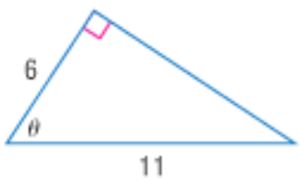
sin θ = ______ cos θ = ______ tan θ = ______

csc θ = ______ sec θ = ______ cot θ = ______

= ______ = ______ = ______

<u>Example 1</u>: Find Trigonometric Values
Find the values of the six trigonometric functions for angle θ.

a)



sin θ =

cos θ =

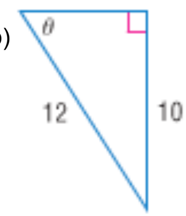
tan θ =

csc θ =

sec θ =

cot θ =

b)



sin θ =

cos θ =

tan θ =

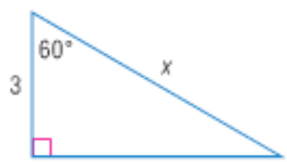
csc θ =

sec θ =

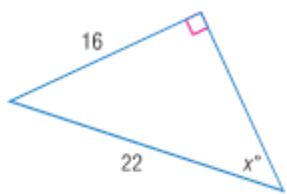
cot θ =

<u>Example 2</u>: Find a Missing Side Length OR Missing Angle Measure of a Right Triangle
Write an equation involving sin, cos, or tan that can be used to find *x*. Then solve the equation. Round measures of sides to the nearest tenth and angles to the nearest degree.

a)

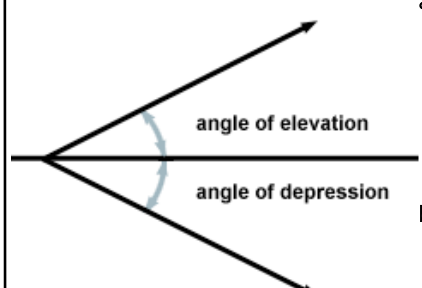


b)



c)

<u>Example 3</u>: Using Angle of Elevation/Depression



a) A plane is flying at an altitude of 12,000 m. From the pilot, the angle of depression to the airport is 32º. How far is the tower from the plane?

b) A ramp for unloading a moving truck has an angle of elevation of 32º. If the top of the ramp is 4 feet above the ground, estimate the length of the ramp.

DAFFYNITION DECODER

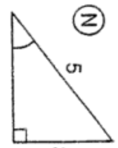
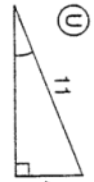
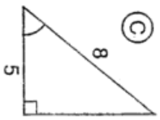
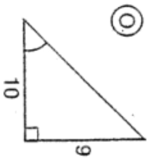
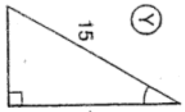
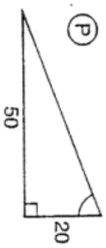
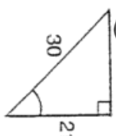
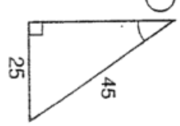
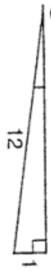
EUROPE:

30°	42°	21°	24°	74°	2°	21°	24°	37°	49°	2°	42°	17°	32°	5°	2°

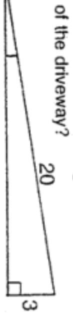
UNDERGROUND GARAGE:

46°	5°	9°	9°	28°	2°	42°	7°	46°	5°	9°	9°	7°	51°	5°	24°	68°	34°	2°

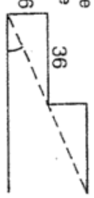
TO DECODE THE TWO DAFFYNITIONS ABOVE: For the first nine exercises, find the measure of the angle indicated. For the remaining exercises, find the angle measure needed to solve the problem. Round to the nearest degree. Each time the answer appears in the code, write the letter of the exercise below it.

(N) 
(U) 
(C) 
(O) 
(Y) 
(P) 
(W) 
(I) 
(A) 

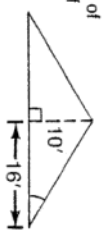
(L) A driveway is built on an incline so that it rises 3 m over a distance of 20 m. What is the angle of elevation of the driveway?



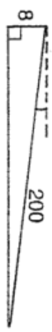
(R) Each step of a stairway rises 16 cm for a tread width of 36 cm. What angle does the stairway make with the floor?



(B) A roof is constructed as shown in the diagram. Find the pitch (angle of elevation) of the roof.



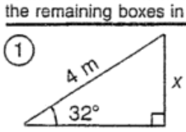
(T) A train decreases its altitude by 8 m when traveling along 200 m of track. Find the angle of depression of the track.



OBJECTIVE 5–d: To use trigonometric ratios to find measures of angles of right triangles.

What Do They Call the Big Grass Field on an Orbiting Satellite?

For the first eight exercises, find the length x. For the remaining exercises, find the length needed to solve the problem. Round each answer to the nearest tenth. Cross out each box that contains a correct answer. When you finish, write the letters from the remaining boxes in the spaces at the bottom of the page.

(1) 
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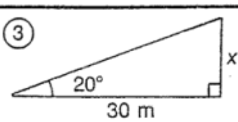
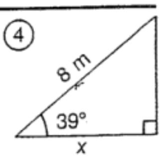
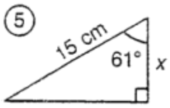
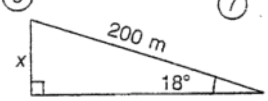
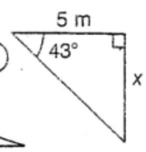
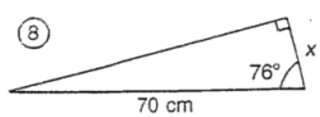
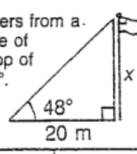
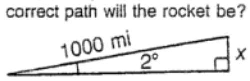
(3) 
(4) 
(5) 
(6) 
(7) 
(8) 

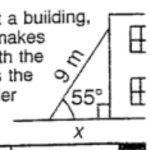
(9) At a point 20 meters from a flagpole, the angle of elevation of the top of the flagpole is 48°. How tall is the flagpole?



(10) If a rocket flies 2° off course for 1000 miles, how far from the correct path will the rocket be?



(11) As it leans against a building, a 9-meter ladder makes an angle of 55° with the ground. How far is the bottom of the ladder from the base of the building?



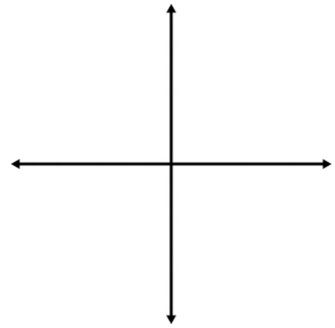
TH 4.7 m	AP 5.4 m	ET 5.2 m	E 2.1 m	AR 23.5 m	UN 6.2 m	A 22.2 m	KI 28.7 mi	SS 61.8 m
RU 18.5 cm	NS 3.2 m	TO 7.3 cm	P 63.6 m	UP 34.9 mi	A 15.3 cm	KY 10.9 m	NI 16.9 cm	CE 17.1 cm

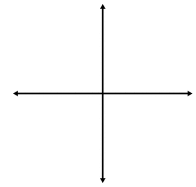
OBJECTIVE 5–c: To use trigonometric ratios to find lengths of sides of right triangles.

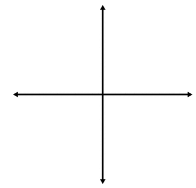
13.2 Angles and Angle Measures

Angle Measurement

On a coordinate plane, an angle may be generated by the rotation of two rays that share a fixed endpoint at the origin. One ray, called the **initial side** of the angle, is fixed along the positive x-axis. The other ray, called the **terminal side** of the angle, can rotate about the center. An angle positioned so that its vertex is at the origin and its initial side is along the positive x-axis is said to be in **standard position**.





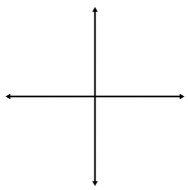


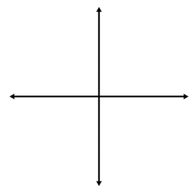
Example 1: Draw an Angle in Standard Position
Draw an angle with the given measure in standard position.

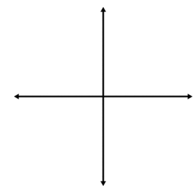
a) 70º

b) 450º

c) -110º







Another unit used to measure angles is a **radian**. As with degrees, the measure of an angle in radians is positive if its rotation is counterclockwise. The measure is negative if the rotation is clockwise.

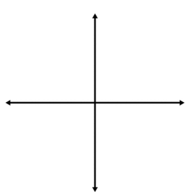
To change angle measures from radians to degrees or vice versa, use the following:

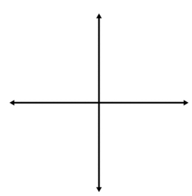
- To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180^\circ}{\pi \text{ radians}}$.
- To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180^\circ}$.

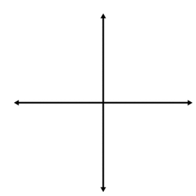
d) $3\pi/4$

b) $-\pi/6$

c) $19\pi/3$







Example 2: Conversions & Coterminal Angles
Rewrite each degree measure in radians and each radian measure in degrees. Then find one positive coterminal angle and one negative coterminal angle.

a) 15º

b) 425º

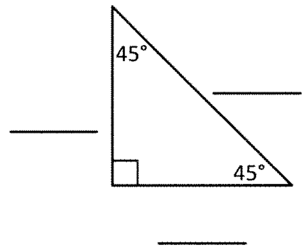
c) $\pi/3$

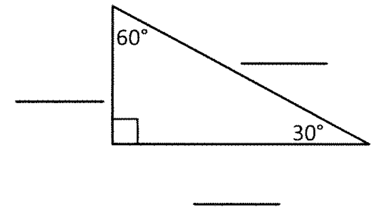
d) $-\pi/4$

Name:____Period:_______

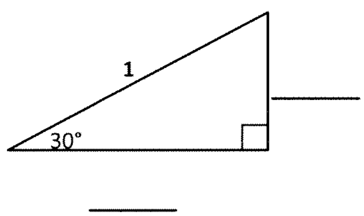
The Unit Circle

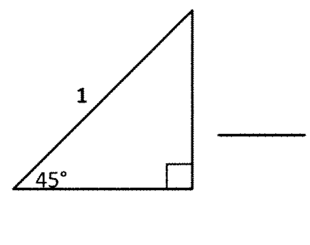
- We're going to be using special right triangles, so first, remind yourself of the patterns of 45-45-90 triangles and 30-60-90 triangles:

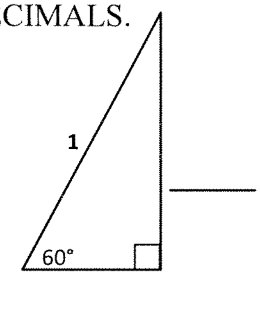




- Use special right triangles to fill in the lengths of the missing sides. NO DECIMALS.







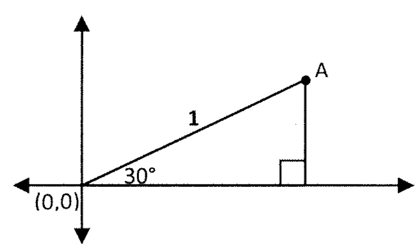
- Now use what you know about trigonometry to write the following sine and cosine ratios for the angles shown in these triangles. SIMPLIFY THOSE RATIOS!

sin 30° = sin 45° = sin 60° =

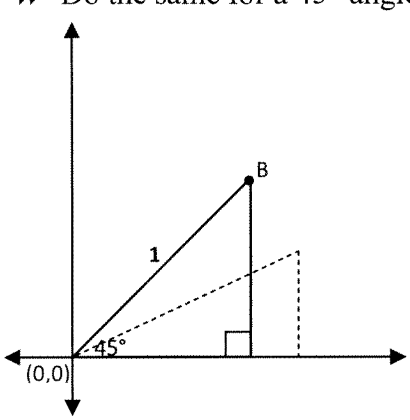
cos 30° = cos 45° = cos 60° =

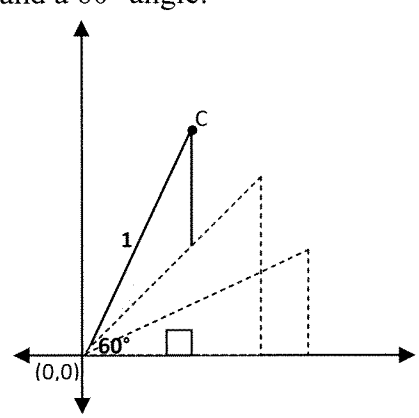
- Next we'll take one of those triangles and put it on a coordinate plane:

What would the *x*- and *y*- coordinates of point *A* (formed by a 30° angle) be? *(Hint: Use the lengths of the sides of the triangle.)*

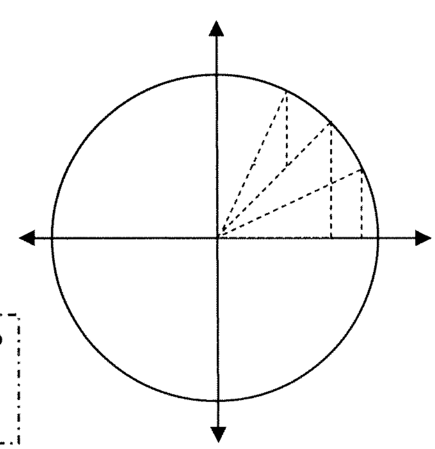


- Do the same for a 45° angle and a 60° angle:





If you look at the three points we have traced out so far, they all lie on a circle centered at the origin.



What is the radius of this circle?

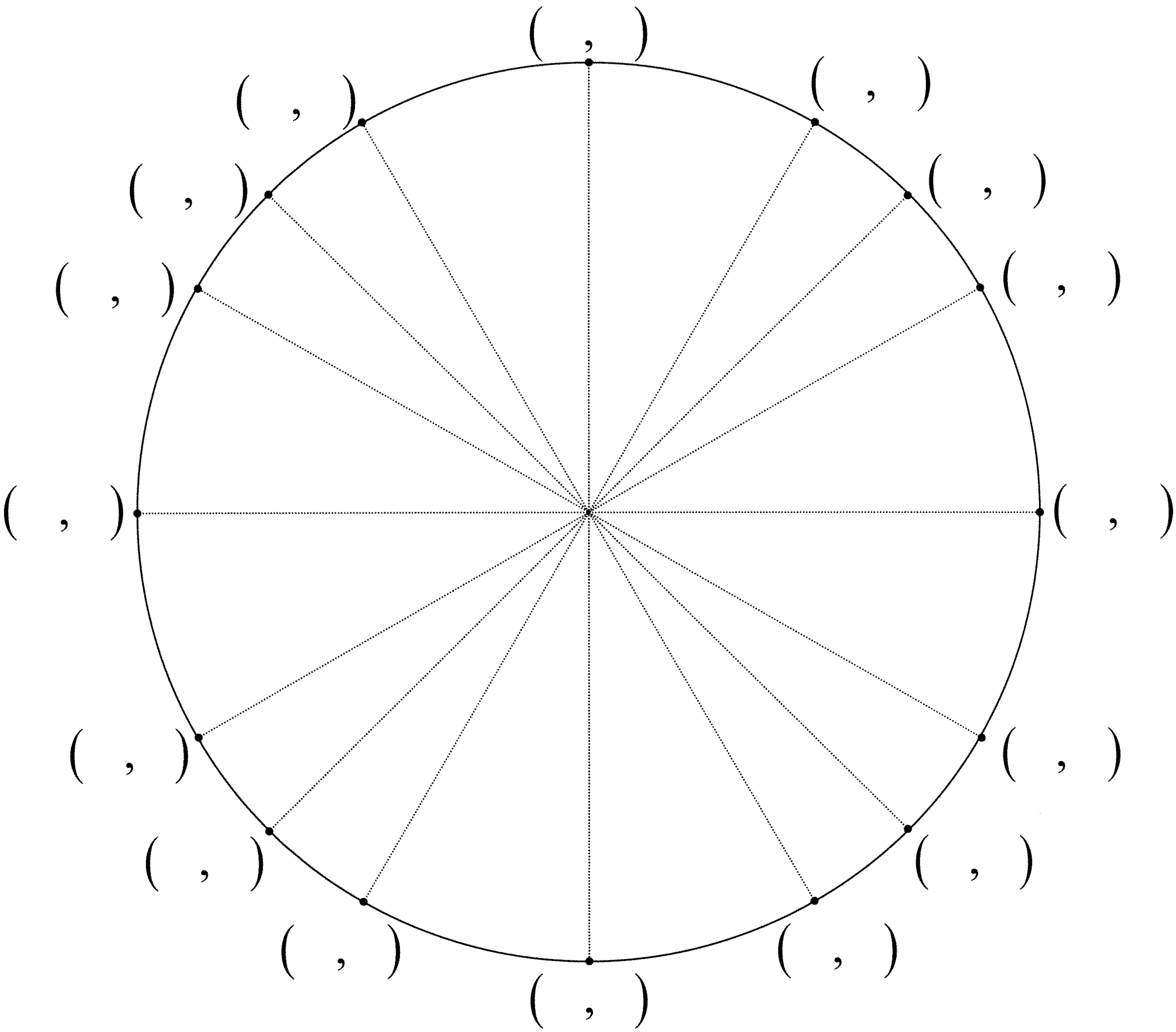
The circle is known as the **unit circle**, since its radius is one unit. It is an integral part of trigonometry.

You will cut out the triangles on the next page to help fill in the coordinates of this unit circle. Use what you know about reflections to help!

HINTS FOR QUADRANT II:
The angles in quadrant II are 120°, 135°, and 150°.

Look at the 120° point and the 60° point.
What should be true about their *y*-coordinates?

What should be true about their *x*-coordinates?

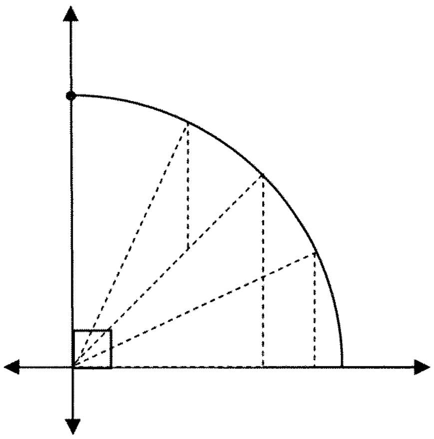


⏩ Use the work you did with the triangles on the front page to fill in the following table:

Angle measure	Coordinate on unit circle	Value of sine	Value of cosine
30°			
45°			
60°			

What relationship do you notice between the coordinate on the unit circle and the values of sine and cosine?

⏩ Look at the point shown below, which corresponds to 90°.



What are the coordinates of this point?

So what is the value of sin 90°?

What is the value of cos 90°?

What would be the coordinates of the point that corresponds to 0°?

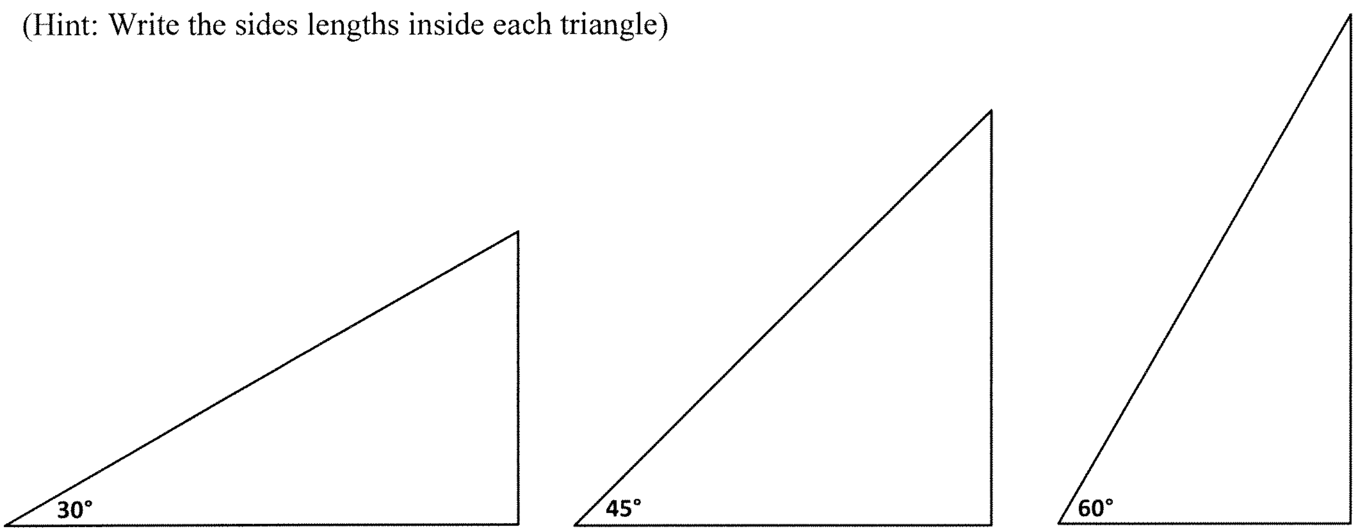
What is sine of 0°?

What is cosine of 0°?

⏩ More practice: *Use the unit circle you just made – your answers should be exact, NO DECIMALS.*

1. sin 120° =
2. cos 180° =
3. cos 135° =
4. sin 315° =
5. cos 240° =
6. sin 330° =

(Hint: Write the sides lengths inside each triangle)



The unit circle is powerful because it can be used to solve trigonometric equations with exact answers rather than decimals that have to be rounded.

For example:

$$\sin 45° = \frac{6}{x}$$

From the unit circle,

$\sin 45° = \frac{\sqrt{2}}{2}$

$$\frac{\sqrt{2}}{2} = \frac{6}{x} \qquad \textit{Substitute}$$

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{12}{\sqrt{2}} \qquad \textit{Cross-multiply} \quad \textit{Divide to isolate x}$$

$$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2} \qquad \textit{Simplify}$$

More importantly, the unit circle lets you solve these trig equations **without** a calculator, which you will often be expected to do in future math classes.

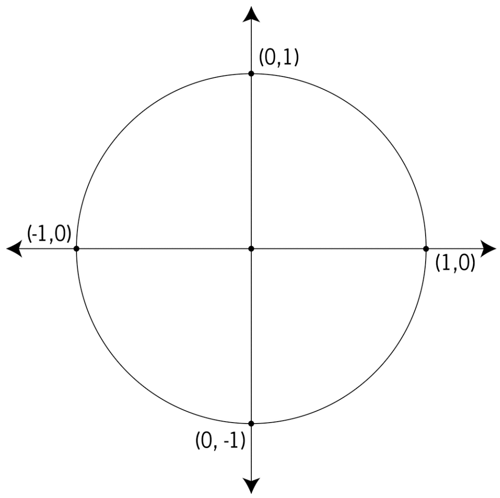
Solve:

$\cos 30° = \frac{x}{18}$ $\qquad$ $\sin 315° = \frac{20}{n}$ $\qquad$ $\cos 210° = \frac{5}{m}$

You should either memorize the unit circle or be able to use triangles to recreate it at a moment's notice.

13.3 Trigonometric Functions of General Angles

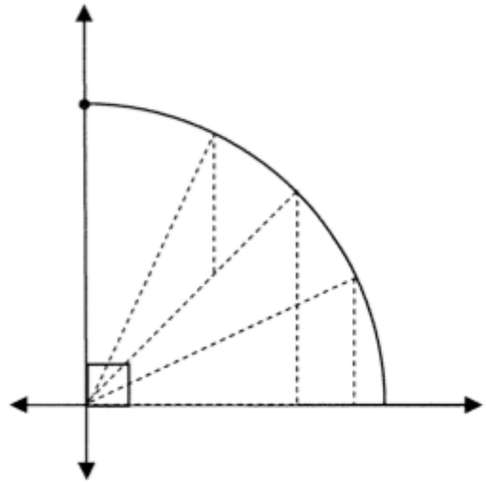
If you know the **quadrants** and **the terminal points of the first quadrant**, you really know the entire circle! LET'S REVIEW!



QUADRANTS

A___ S__________ T__________ C__________!

This tells you when each trig. function and their reciprocal functions are **positive**.

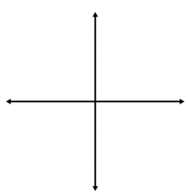


FIRST QUADRANT

Knowing the first quadrant will help you find exact values around the entire unit circle. You just need to know how to find the **reference angle**.

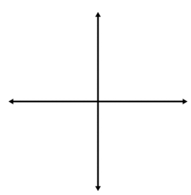
Example 1: Find the Reference Angle for a Given Angle
Sketch each angle. Then find its reference angle.

a) 235^{o}

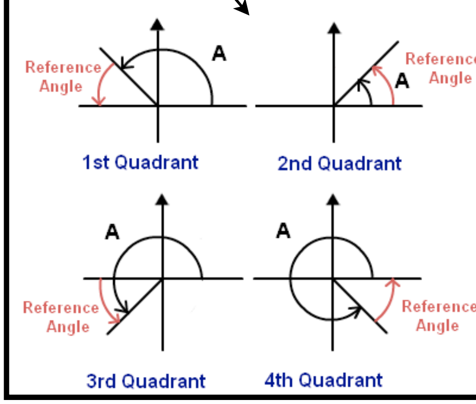


ref. angle = ______

b) $-7\pi/4$



ref. angle = ______



Now that you know how to find reference angles, you can use them to find the values of trigonometric functions of any angle. Just follow these steps.

TO FIND THE EXACT VALUE OF A TRIGONOMETRIC FUNCTION...

Step 1: ____

Step 2: ____

Step 3: ____

Example 2: Use a Reference Angle to Find a Trigonometric Value
Find the exact value of each trigonometric function.

a) sin 300°

ref. angle = ______

Q_____

exact value: __________

b) cos 135°

ref. angle = ______

Q_____

exact value: __________

c) cos 180°

ref. angle = ______

Q_____

exact value: __________

d) tan 5π/6

ref. angle = ______

Q_____

exact value: __________

e) csc 5π/3

ref. angle = ______

Q_____

exact value: __________

f) sec 7π/4

ref. angle = ______

Q_____

exact value: __________

If you know the quadrant that contains the terminal side of θ in standard position and the exact value of one of the trigonometric functions of θ, you can find the values of the other trigonometric functions of θ using the function definitions.

Example 3: Quadrant and One Trigonometric Value of θ
Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

a) tan θ = -2/3, Quadrant IV

b) cos θ = -1/2, Quadrant II

c) cot θ = - $\sqrt{2}/2$, Quadrant IV

RIGHT TRIANGLE TRIGONOMETRY - WORD PROBLEMS **Name ___________________________ Period ___**

For each problem, do the following:

1) Draw a picture. (Right triangle)
2) Label the given parts.
3) Set up the trig. ratios and solve.

1. A damsel is in distress and is being held captive in a tower. Her knight in shining armor is on the ground below with a ladder. When the knight stands 15 feet from the base of the tower and looks up at his precious damsel, the angle of elevation to her window is 60 degrees. How long does the ladder have to be?

2. A 12-meter flagpole casts a 9-meter shadow. Find the angle of elevation of the sun.\

3. Suppose you're flying a kite, and it gets caught at the top of the tree. You've let out all 100 feet of string for the kite, and the angle that the string makes with the ground is 75 degrees. Instead of worrying about how to get your kite back, you wonder. "How tall is that tree?"

4. A submersible traveling at a depth of 250 feet dives at an angle of 15 degrees with respect to a line parallel to the water's surface. It travels a horizontal distance of 1500 feet during the dive. What is the depth of the submersible after the dive?

5. A fire department's longest ladder is 110 feet long, and the safety regulation states that they can use it for rescues up to 100 feet off the ground. What is the maximum safe angle of elevation for rescue ladder?

6. Brothers Bob and Tom buy a tent that has a center pole 6.25 feet high. If the sides of the tent are supposed to make a 50 degree angle with the ground, how wide is the tent?

7. A swimming pool is 30 meters long and 12 meters wide. The bottom of the pool is slanted so that the water depth is 1.3 meters at the shallow end and 4 meters at the deep end. Find the angle of depression of the bottom of the pool.

8. In rhombus ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ meet at point E. If the measure of angle DAB is 46 degrees, find the length of $\overline{EB}$.

9. The tallest television transmitting tower in the world is in North Dakota, and it is 2059 feet tall. If you are on a level ground exactly 5280 feet (one mile) from the base of the tower, what is your angle of elevation looking up at the top of the tower?

10. Ophelia Payne is walking to her office building which she knows is 150 feet high. The angle to the top of the building from her current location is 6 degrees. How much further does she need to walk?

11. A communications tower is built on top of a building with the following specifications: from a point 200 meters from the base of the building, the angle of elevation to the top of the building is 23.6 degrees and the angle of elevation to the top of the tower is 15.9 degrees. Find the height of the tower.