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12.1 The Counting Principle

DEFINITIONS

- **Fundamental Counting Principle** - if event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in m x n ways.
- **independent events** - if the outcome of an event does not affect the outcome of another event, the two events are independent
- **dependent events** - if the outcome of an event does affect the outcome of another event, the two events are dependent

Examples:

a) A cafeteria offers drink choices of water, coffee, juice, and milk and salad choices of pasta, fruit, and potato. How many different combinations of drink and salad are possible?

b) Dane is renting a tuxedo for prom. Once she has chosen his jacket, he must choose from three types of pants and six colors of vests. How many different ways can he select his attire for prom?

c) If a garage door opener has a 10-digit keypad and the code to open the door is a 4-digit code, how many codes are possible?

d) Each player in a board game uses one of six different pieces. If four players play the game, how many different ways could the players choose their game pieces?

e) The Murray family is choosing from a trip to the beach or a trip to the mountains. The family can select transportation from a car, train, or plane. How many ways can the family select a destination followed by a means of transportation?

f) For a college application, Macawi must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays?

g) How many answering machine codes are possible if the code is just two digits?

h) A sandwich menu offers customer a choice of white, wheat, or rye bread with one spread chosen from butter, mustard, or mayonnaise. How many different combinations of bread and spread are possible?

12.2 Permutations and Combinations

Permutations
When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**. In a permutation, the *order* of the objects is very important.

The number of permutations of n distinct objects taken r at a time is given by $P(n,r)=\dfrac{n!}{(n-r)!}$

Permutations with Repetitions
The number of permutations of n objects of which p are alike and q are alike is $\dfrac{n!}{p!q!}$

Combinations
An arrangement or selection of objects in which *order* is NOT important is called a **combination**.

Then number of combinations of n distinct objects taken r at a time is given by $C(n,r)=\dfrac{n!}{(n-r)!r!}$

Example 1: Permutations VS. Combinations
Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

a) A newspaper has nine reporters available to cover four different stories. How many ways can the reporters be assigned to cover the stories?

b) How many different ways can the letters of the word DECIDED be arranged?

c) A family with septuplets assigns different chores to the children each week. How many ways can three children be chosen to help with the laundry?

d) How many different ways can the letters of the word BANANA be arranged?

e) Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people to go?

f) Eight people enter the Best Pie contest. How many ways can blue, red, and yellow ribbons be awarded?

Example 2: Multiple Events
*In more complicated situations, you may need to multiply combinations and/or permutations.

a) Six cards are drawn from a standard deck of cards. How many hands consist of two hearts and four spades?

b) How many five-card hands consist of five cards of the same suit?

12.3 Probability

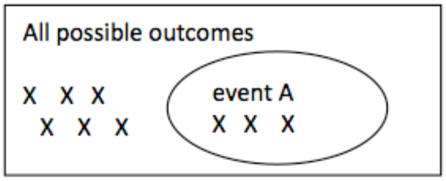
In probability, a desired outcome is called a **success**; any other outcome is called a **failure**.

Probability of Success and Failure	If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows. $P(S) = \frac{s}{s+f}$ and $P(F) = \frac{f}{s+f}$.

There are two types of probability we will be learning today:
- **theoretical probability**
- **experimental probability**

The **<u>Theoretical Probability</u>** that an event A will occur is:

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$



In this diagram, there are 9 total outcomes and 3 are in A so $P(A) = \frac{3}{9} = \frac{1}{3}$ (Yes, you are expected to reduce fractions!)

However, theoretical probability is not always convenient to find. In this case, finding the **<u>Experimental Probability</u>** may be an option. Experimental probability is found by performing an experiment, taking a survey or looking at the history of an event.

<u>Example 1</u>: Theoretical Probability

a) Slips of paper numbered 1 through 12 are placed in a hat. You have an equally likely chance of choosing any of these integers. Find the probability of each given event.

(1) an even number is chosen **(2)** a prime number is chosen **(3)** a factor of 48 is chosen

b) A cookie jar contains 4 chocolate chip cookies, 8 peanut butter cookies, 5 sugar cookies and 3 oatmeal cookies. Find the probability of each given event.

(1) P(chocolate chip) **(2)** P(sugar) **(3)** P(not oatmeal)

<u>Example 2</u>: Experimental Probability

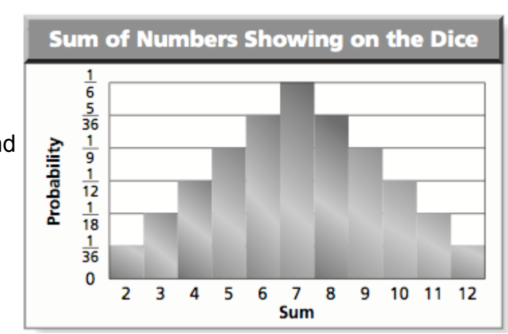
In 1998 a survey asked internet users for their ages. The results are summarized in the table below.
a) Find the experimental probability that a randomly selected internet user is at most 20 years old.
b) Find the experimental probability that a randomly selected internet user is at least 41 years old

Age	Number of Users
Under 21	1636
21 – 40	6617
41 – 60	3693
61 – 80	491
Over 80	6

<u>Example 3</u>: Probability Distribution

Suppose two dice are rolled. The table and the relative frequency histogram show the distribution of the sum of the numbers rolled.

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$



a) Use the graph to determine which outcomes are the second most likely. What are their probabilities?

b) Use the table to find P(S = 4). What other sum has the same probability?

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12.3 Probability

Find the probability for each of the following. Work must be shown for credit!

1. You earned an A on 4 of your last 7 quizzes. What is the probability that you will get an A on your next quiz?

2. You roll a six-sided die whose sides are numbered from 1 through 6. What is the probability of each?
   a) P(rolling a 5) b) P(rolling an even number)

3. A jar contains 5 red marbles, 3 green marbles, 4 yellow marbles, and 2 blue marbles. Calculate the probability of the following events:
   a) P(yellow) b) P(red)

4. You have an equally likely chance of choosing any integer from 1 through 20. Find the probability of the given event.
   a) P(an odd number chosen) b) P(a number less than 7 is chosen)

   c) P(a perfect square is chosen) d) P(a prime number is chosen)

   e) P(a multiple of 3 is chosen) f) P(a factor of 240 is chosen)

5. A jar contains 2 red marbles, 3 blue marbles, and 1 green marble. Find the probability of randomly drawing the given type of marble.
   a) P(a blue or a green marble) b) P(a red or a blue marble) c) P(a green or a red marble)

6. The table shows how students in Mr. Gaylor's class fared on their first driving test. Some took a class to prepare, while others did not.
   a) Find the probability that Paige took the class.

   b) Find the probability that Elizabeth did not take the class.

	Class	No Class
Passed	64	48

7. The number of students who have attended a football game at Central High School is listed below. Find the probability that a student who has attended a game is a junior or a senior.

Class	Freshman	Sophomore	Junior	Senior
Attended	48	90	224	254

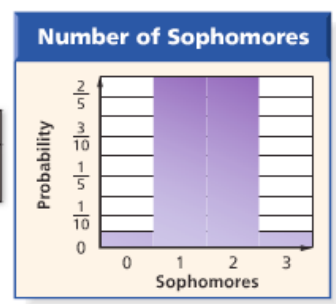
8. Three students are selected at random from a group of 3 sophomores and 3 juniors. The table and relative-frequency histogram show the distribution of the number of sophomores chosen. Find each probability.

   a) P(0 sophomores) b) P(1 sophomore)

   c) P(2 sophomores) d) P(3 sophomores)

   e) P(2 juniors) f) P(1 junior)

Sophomores	0	1	2	3
Probability	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$



9. Use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) recently. If a student taking the test were randomly selected, find each probability. Express as decimals rounded to the nearest thousandth.

   a) P(math or statistics) b) P(biological sciences) c) P(physical sciences)

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

12.4 Multiplying Probabilities

KEY CONCEPTS

Probability of Two Independent Events
If two events, A and B, are independent, then the probability of both events occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Probability of Two Dependent Events
If two events, A and B, are dependent, then the probability of both events occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$$

Example 1: Two Independent Events

a) At a promotional event, a radio station lets visitors spin a prize wheel. The wheel has 10 sectors of the same size for posters, 6 for T-shirts, and 2 for concert tickets. What is the probability that two consecutive visitors will win posters?

b) Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both coins he selects are dimes?

Example 2: Three Independent Events

a) In a state lottery game, each of three cages contains 10 balls. The balls are each labeled with one of the digits 0-9. What is the probability that the first two balls drawn will be even and that the third will be prime?

b) When three dice are rolled, what is the probability that the first two show a 5 and the third shows an even number?

Example 3: Two Dependent Events

a) The host of a game show is drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, 6 show *television*, 3 show *vacation*, and 1 shows *car*. If the host draws the chips at random and does not replace them, find the probability that the host draws two televisions.

b) The next week, the host of the game show draws from a bag of 20 chips, of which 11 say *computer*, 8 say *trip*, and 1 says *truck*. If chips are drawn at random and without replacement, find the probability of drawing a computer, then a truck.

Example 4: Three Dependent Events

a) Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart, another heart, and a spade in that order.

b) Find the probability of drawing three diamonds.

12.5 Adding Probabilities

KEY CONCEPTS

Probability of Mutually Exclusive Events
If two events, A and B, are mutually exclusive, then the probability that A or B occurs is the sum of their probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$

Probability of Inclusive Events
If two events, A and B, are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Examples: Mutually Exclusive and Inclusive Events
Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

a) One teacher must be chosen to supervise a senior class fundraiser. There are 12 math teachers, 9 language arts teachers, 8 social studies teachers, and 10 science teachers. If the teacher is chosen at random, what is the probability that the teacher is either a language arts teacher or a social studies teacher?

b) There are 2400 subscribers to an Internet service provider. Of these, 1200 own Brand A computers, 500 own Brand B, and 100 own both A and B. What is the probability that a subscriber selected at random owns either Brand A or Brand B?

c) Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at fandom from this stack, what is the probability that it is a heart or a club?

d) Sixty plastic discs, each with one of the numbers from 1 to 60, are in a bag. LaTanya will win a game if she can pull out any disc with a number divisible by 2 or 3. What is the probability that LaTanya will win?

e) A card is drawn from a standard deck of cards. What is the probability that it is a 5 or a spade?

f) A jar of change contains 5 quarters, 8 dimes, 10 nickels, and 19 pennies. If a coin is pulled from the jar at random, what is the probability that it is a nickel or a dime?

Standard 52-card deck

K ♠	K ♣	K ♦	K ♥
Q ♠	Q ♣	Q ♦	Q ♥
J ♠	J ♣	J ♦	J ♥
10 ♠	10 ♣	10 ♦	10 ♥
9 ♠	9 ♣	9 ♦	9 ♥
8 ♠	8 ♣	8 ♦	8 ♥
7 ♠	7 ♣	7 ♦	7 ♥
6 ♠	6 ♣	6 ♦	6 ♥
5 ♠	5 ♣	5 ♦	5 ♥
4 ♠	4 ♣	4 ♦	4 ♥
3 ♠	3 ♣	3 ♦	3 ♥
2 ♠	2 ♣	2 ♦	2 ♥
A ♠	A ♣	A ♦	A ♥

Outcomes of Rolling Two Number Cubes