Chapter _____: ____

Name ____ Period __________

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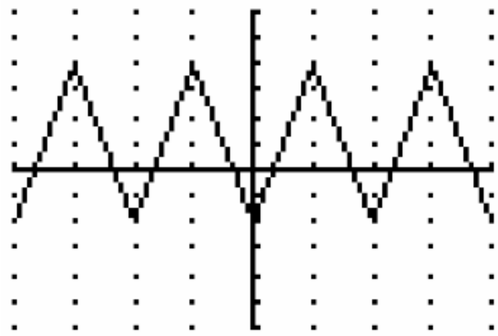
13.6 Circular Functions (Day 1)

A period function or graph is a function or graph that repeats the same pattern over and over again. We will use the following periodic graph to define the characteristics of a periodic graph or function.



cycle -

period -

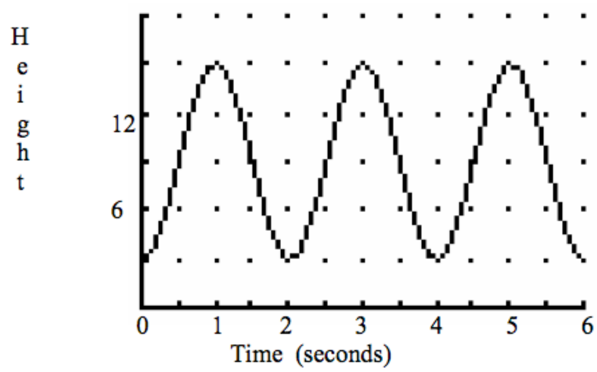
amplitude -

center line axis -

frequency -

Many real-world situations have characteristics that can be modeled with a periodic function:

A little girl is jumping on a trampoline. The graph below shows how high her feet are off the ground as a function of time.



Find each of the following:

a) the cycle

b) the period

c) the amplitude

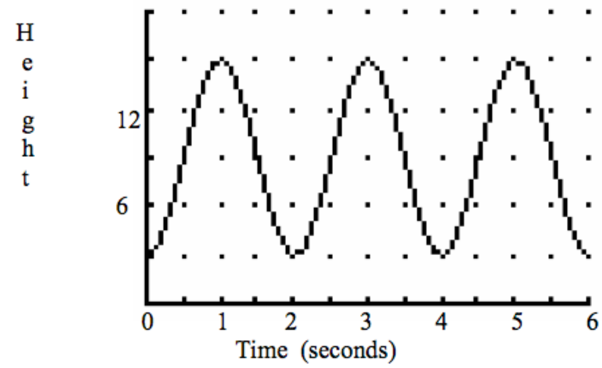
d) the equation of the center line

e) the frequency

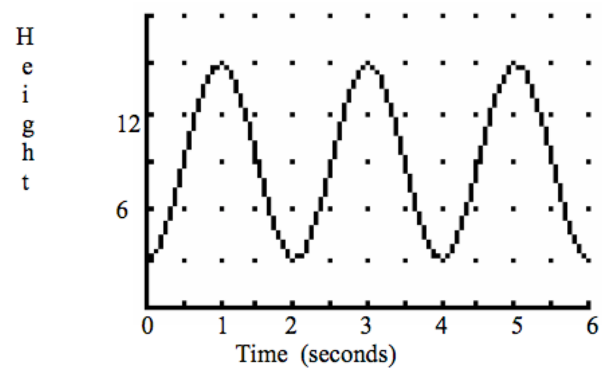
13.6 Circular Functions (Day 1)

Redraw the graph using the given information:

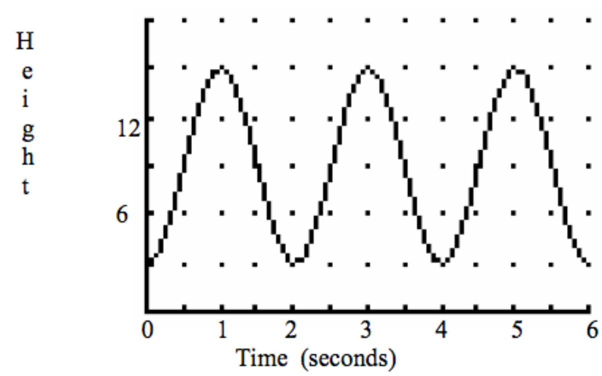
a) a vertical translation of 2



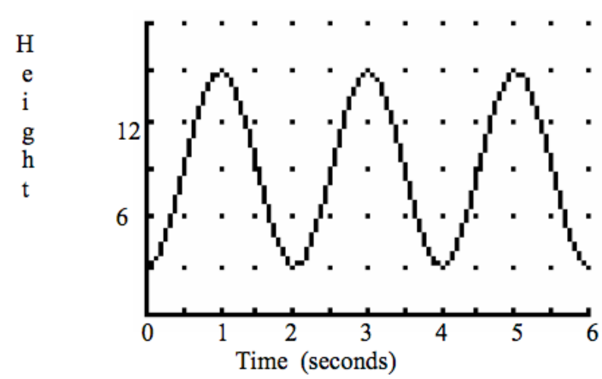
b) the period was doubled



c) a horizontal translation of 1

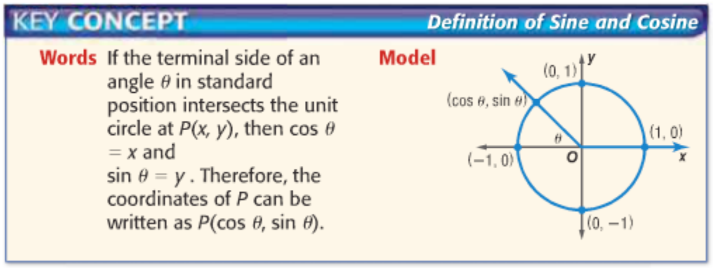


d) a vertical stretch by a factor of 2



13.6 Circular Functions (Day 2)

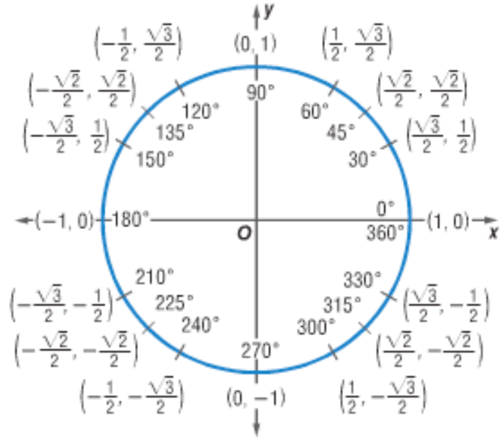
The exact values of the sine and cosine functions for specific angles are summarized using the definition of sine and cosine on the unit circle at the right.



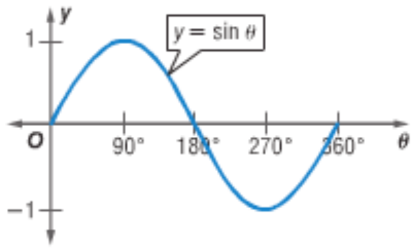
KEY CONCEPT *Definition of Sine and Cosine*

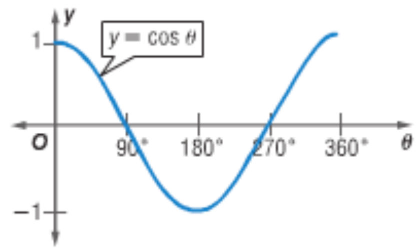
Words If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$. Therefore, the coordinates of P can be written as $P(\cos\theta, \sin\theta)$.

Model

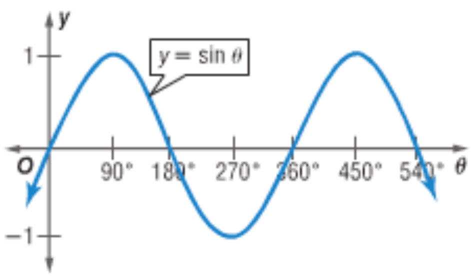


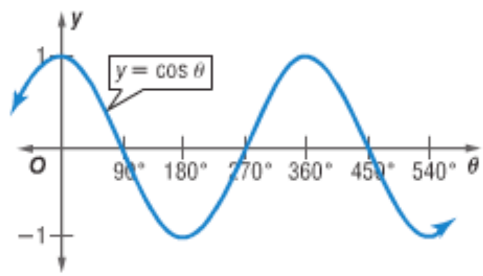
This same information is presented on the graphs of the sine and cosine functions below, where the horizontal axis shows the values of θ and the vertical axis shows the values of sin θ or cos θ.





Notice in the graph above the values of sine for the coterminal angles 0º and 360º are both 0. The values of cosine for these angles are both 1. Every 360º or 2π radians, the sine and cosine functions repeat their values. So, we can say that the sine and cosine functions are **periodic**, each having a **period** of 360º or 2π radians.



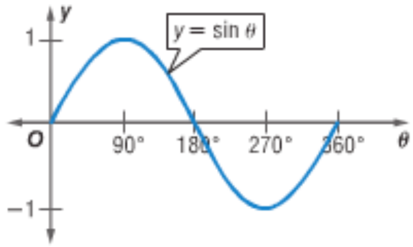


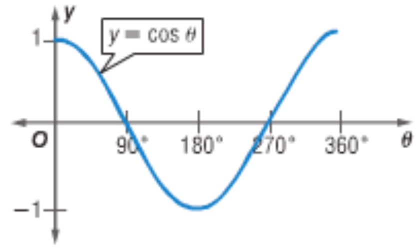
Let's complete the characteristics for the two functions for $0^\circ \le x \le 360^\circ$.

	y = sin x	y = cos x
Amplitude		
Period		
Center Line		
x-intercepts		
y-intercepts		
Increasing		
Decreasing		

13.6 Circular Functions (Day 2)

Classwork:





Tell whether each statement below describes a characteristic of the *sine function*, the *cosine function*, both functions or neither function.

a) The function has an amplitude of 1

b) The function passes through (0,0)

c) The function is increasing on the interval $0^o \leq x \leq 90^o$

d) The period of the function is 180^o

e) The center line axis of the function is y = 0

f) The horizontal intercepts occur at multiples of 180^o

g) The horizontal intercepts occur at multiples of 90^o

h) The range of the function is $0 \leq y \leq 1$

i) The maximum values of the function occur at multiples of 360^o

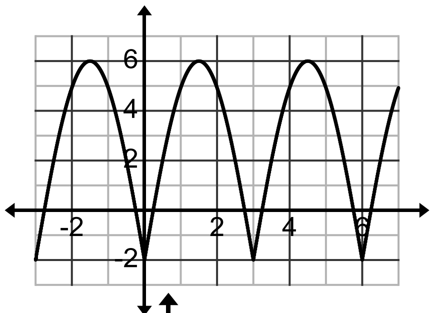
j) The minimum value of the function is -1

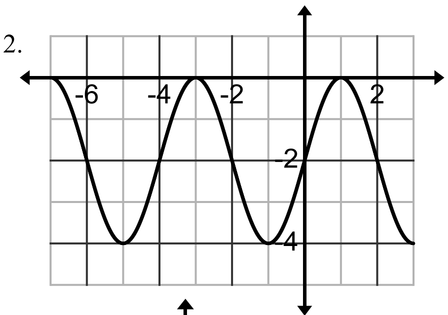
k) The function decreases on the interval $0^o \leq x \leq 180^o$

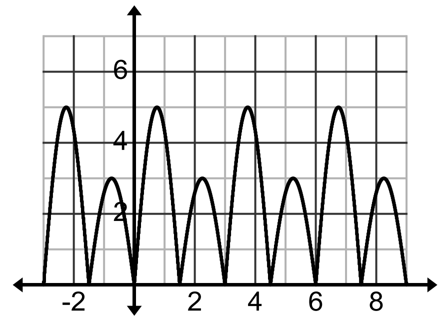
Based on what we have learned, can you find the value for which sin x = cos x on $-360^o \leq x \leq 360^o$?

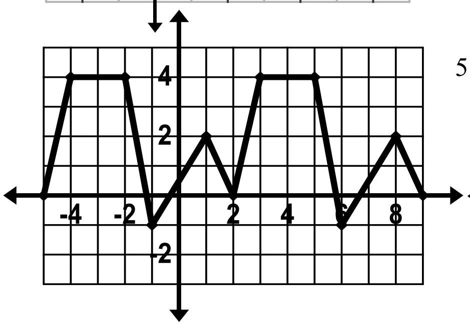
Algebra 2CP
Trig Unit Part II Worksheet

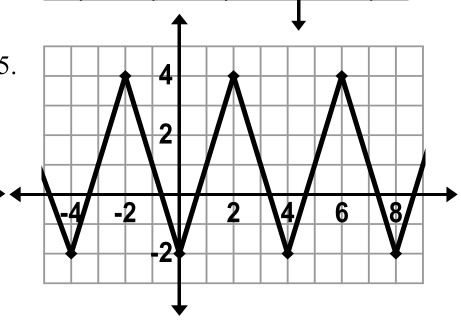
For each of the five graphs below a) draw one cycle b) find the period c) write the equation of the center line axis d) find the amplitude e) find the frequency

1.


2.


3.


4.


5.


6. Redraw Graph 1 with a vertical translation of -2.
7. Redraw Graph 2 with a period that is doubled.
8. Redraw Graph 3 with a horizontal translation of 2.
9. Redraw Graph 3 with a period that is cut in half.

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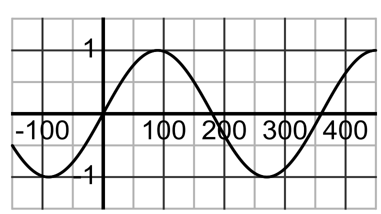
10. For each of the following, find the domain values $0° < x < 360°$ for which the graph of
   a) $y = \sin(x)$ decreases from 1 to 0
   b) $y = \cos(x)$ decreases from 1 to 0
   c) $y = \sin(x)$ increases from -1 to 0
   d) $y = \cos(x)$ increases from -1 to 0
   e) $y = \sin(x)$ increases from 0 to 1
   f) $y = \cos(x)$ increases from 0 to 1
   g) $y = \sin(x)$ decreases from 0 to -1
   h) $y = \cos(x)$ decreases from 0 to -1

11. Tell whether each of the following statements describes a characteristic of the sine function, the cosine function, both functions or neither function.

   a) The function increases throughout the interval $180° < x < 360°$.
   b) The domain of the function is all real numbers.
   c) The graph crosses the x-axis at multiples of $180°$.
   d) The amplitude of the function is -1.
   e) The function has a period of $180°$.
   f) The function passes through (0,1)
   g) The function is increasing on the interval $0° < x < 90°$.
   h) The center line axis of the function is y = 0.
   i) The maximum value is 1.
   j) The range of the function is $-1 \le y \le 1$.

12. Use the graph of $y = \sin(x)$ to estimate the value of each of the following.
   a) sin(35°) b) sin(115°) c) sin(235°) d) sin(335°)



13. A vertical gear of an old clock makes one counterclockwise revolution every 60 seconds. Suppose there is a catch on the side of the gear that is at its rightmost position at the time $t = 0$ and suppose the vertical position of the catch at this time is called $h = 0$.

   a) If the vertical position of the catch after 5 seconds is $h = 4\,mm$, after how many more seconds will it again be at $h = 4\,mm$?
   b) Name two times during the first 60 seconds that its vertical position will be $h = -4\,mm$.

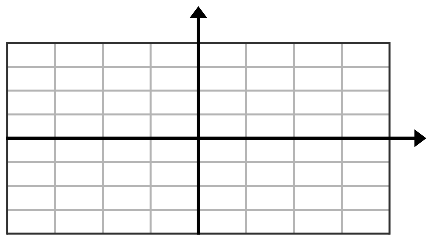
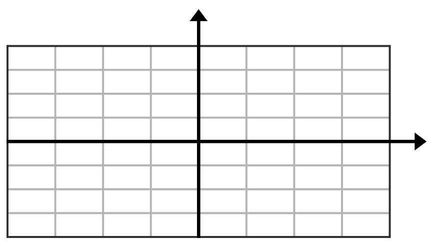
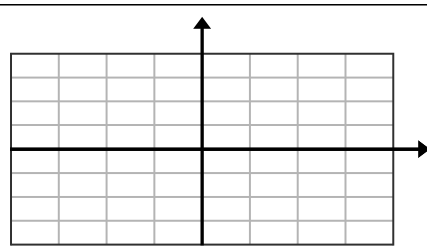
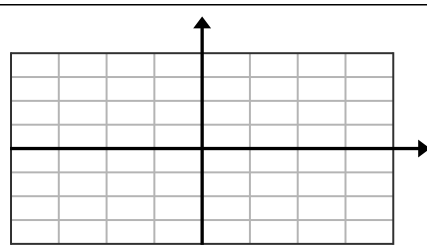
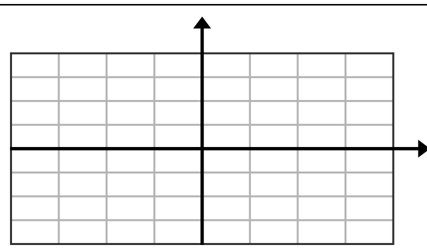
Investigation #1 Name ________________________

The Sine Function: Amplitude Period ________________________

In this lesson you will learn how the constant A affects (or transforms) the graph of $y = A\sin(x)$

1. Use a graphing calculator to graph each of the following functions. All work will be done in degrees, so you must set the mode setting on your calculator to degrees. The suggested window settings are $X\min = -360$, $X\max = 360$, Xscl = 90, $Y\min = -4$, $Y\max = 4$, Yscl =1.

   Be sure to identify the value of A for each graph.

Equation	A	Sketch	Max	Min	Amplitude = $\frac{1}{2}(\max - \min)$	Is the graph increasing or decreasing from 0° to 90°?
$y = 1\sin(x)$						
$y = 2\sin(x)$						
$y = 0.5\sin(x)$						
$y = -2\sin(x)$						
$y = -3.5\sin(x)$						

2. Use your graphs to answer the following questions about $y = A\sin(x)$.

   a. As $|A|$ increases, does the graph become steeper or flatter? ____________
   b. Does the sign of A affect the value of the maximum, minimum, or amplitude? If so, how? ____

      ____
   c. How do the graphs of $y = A\sin(x)$ and $y = -A\sin(x)$ differ? ____________

      ____
   d. Are the graphs of $y = A\sin(x)$ and $y = -A\sin(x)$ symmetric? If so, are they symmetric about the x-axis or the y-axis?____________________
   e. If $A = 2.5$, will the maximum be greater or less than the graph with $A = 1$?

      ____
   f. Suppose you want the maximum value of $y = A\sin(x)$ to be 1.5 and the graph to be increasing from 0° to 90°. What value of A would you choose? Check your answer on the calculator. ____________________
   g. Suppose you want the minimum value of $y = A\sin(x)$ to be between -1.25 and -1.50 and the graph to be decreasing from 0° to 90°. What value of A would you choose? Check your answer on the calculator.

      ____

3. Write formulas for the maximum value, minimum value, and amplitude in terms of the constant A in the equation $y = A\sin(x)$. Remember that A can be either positive or negative.

   Maximum: ____________________
   Minimum: ____________________
   Amplitude: ____________________

4. Explain how the constant A affects (or transforms) the graph of $y = A\sin(x)$.

   ____
   ____

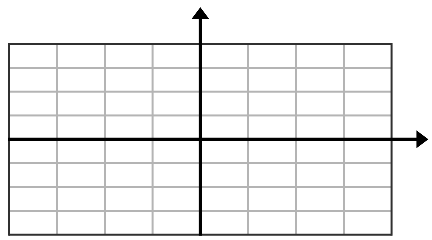
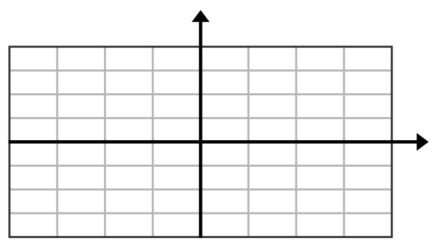
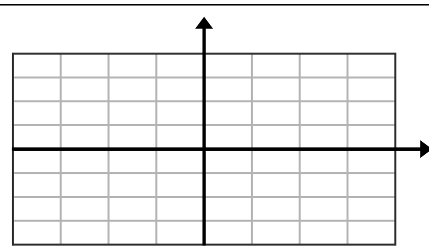
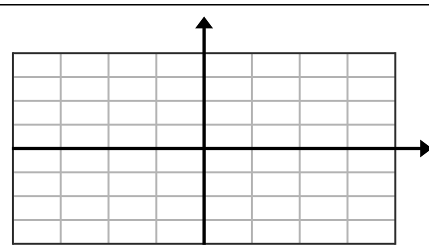
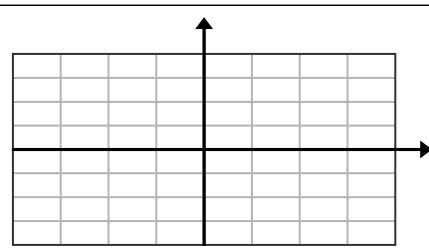
Investigation #2 Name ____________________

The Sine Function: Vertical Shift

In this lesson you will learn how the constants A and D affect (or transform) the graph of $y = A\sin(x) + D$.

1. Use a graphing calculator to graph each of the following functions. All work will be done in degrees, so you must set the mode setting on your calculator to degrees. The suggested window settings are $X\min = -360$, $X\max = 360$, Xscl = 90, $Y\min = -4$, $Y\max = 4$, Yscl =1.

Be sure to identify the values of A and D for each graph.

Equation	A	D	Sketch	Max	Min	$\frac{Max + Min}{2}$
$y = 1\sin(x) + 0$						
$y = 1\sin(x) + 2$						
$y = \sin(x) - 3$						
$y = 2\sin(x) + 2$						
$y = -1\sin(x) + 1$						

2. Use your graphs to answer the following questions about $y = A\sin(x) + D$

a. If the constant D is positive, does the graph shift up or down? __________

b. If the constant D is negative, does the graph shift up or down? __________

c. Write a sentence describing what happens to the graph if we add a non-zero constant D to the equation $y = A\sin(x)$. ____________________

____

d. If you want the graph of $y = A\sin(x)$ to shift 1.5 units above the x-axis, what value of D should you choose? ____________________

e. If a function is periodic, like the sine function, then one-half the sum of the maximum value plus the minimum value is the center line of the function. What does the center line of $y = A\sin(x) + D$ tell you about the graph?

____

f. The graph of $y = 1\sin(x) + 2$ has a new center line because it has been shifted up from the x-axis. What is the equation of the new center line?

____

g. Explain how the constants A and D affect the shape and location of the graph of $y = A\sin(x) + D$. ____________________________

3. State formulas for the vertical shift, center line, maximum value and minimum value in terms of the constants A and D in the equation $y = A\sin(x) + D$.

Vertical Shift: ________________ Center Line: ________________

Maximum: ________________ Minimum: ________________

4. Write equations of the form $y = A\sin(x) + D$ for the maximum, minimum, and vertical shift values given below. The first entry has been completed for you.

Maximum	Minimum	Amplitude	Vertical Shift	Equation
3	1	1	2	$y = 1\sin(x) + 2$
1	-3	2	-1	
2	-1			
3	-1			

5. Write an equation whose graph is a sine curve between the graphs of the equations $y = -1\sin(x) + 2$ and $y = -1\sin(x) + 0.5$. Verify your answer using the calculator. ____________________________

6. Explain how the constant D affects (or transforms) the graph of $y = A\sin(x) + D$.____________________________

____

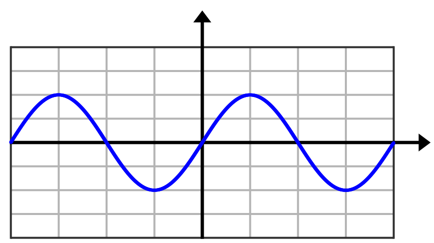
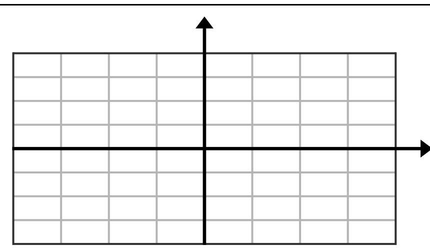
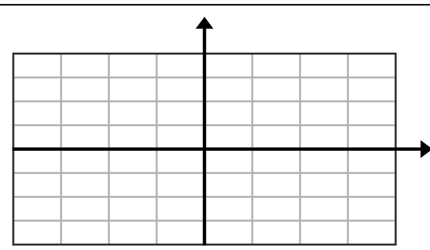
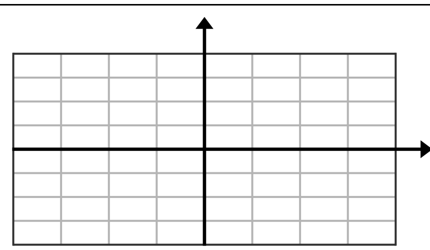
Investigation #3 Name ____________________

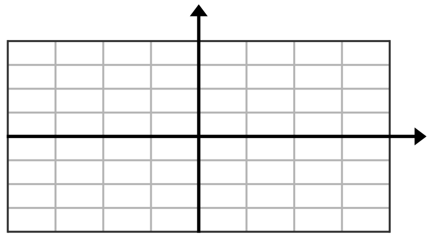
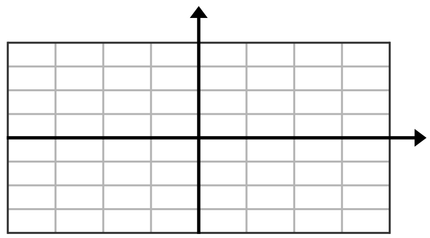
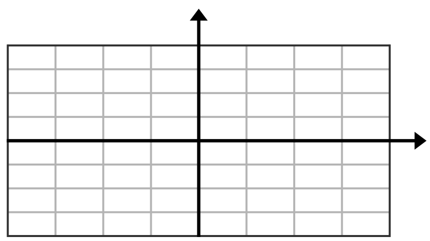
The Sine Function: Phase Shift

In this lesson you will learn how the constant C affects (or transforms) the graph of $y = \sin(x - C)$. Phase shift tells how far (in degrees) the graph has moved in the horizontal direction.

1. Use a graphing calculator to graph each of the following functions. All work will be done in degrees, so you must set the mode setting on your calculator to degrees. The suggested window settings are $X\min = -360$, $X\max = 360$, Xscl = 90, $Y\min = -2$, $Y\max = 2$, Yscl = 0.5. (NOTE: The y scale has changed.) The first one has been done for you.

Be sure to identify the value of C for each graph.

Equation	C	Sketch	Phase Shift	x-intercepts between 0 and 360°
$y = \sin(x)$	0		0°	0°, 180°, 360°
$y = \sin(x - 45)$				
$y = \sin(x - 90)$				
$y = \sin(x - 180)$				

Equation	C	Sketch	Phase Shift	x-intercepts between 0 and 360°
$y = \sin(x - 270)$				
$y = \sin(x + 45)$				
$y = \sin(x + 90)$				

2. Compared to $y = \sin(x)$, have the graphs in Exercise 1 been shifted horizontally or vertically? ______________________________
3. In what direction does the graph shift when $C > 0$? ____________________
4. In what direction does the graph shift when $C < 0$? ____________________
5. Explain why the x-intercepts of $y = \sin(x)$ and $y = \sin(x - 45)$ are different.

   ____
6. Explain the difference between the phase shifts in the graphs $y = \sin(x - 45)$ and $y = \sin(x + 45)$. _____________________________________
7. What is the formula for the phase shift in terms of C? __________________
8. Use what you know about phase shift and period to explain why $y = \sin(x - 360)$ has the same graph as $y = \sin(x)$. ______________________

   ____
9. Write an equation of the form $y = \sin(x - C)$ that has x-intercepts at 60° and 240°. Check your answer using the calculator. ________________________
10. Give a value of C in the equation $y = \sin(x - C)$ that would produce a graph between $y = \sin(x + 120)$ and $y = \sin(x + 45)$. Check your answer using the calculator.____
11. Explain how the constant C affects (or transforms) the graph of $y = \sin(x - C)$.____

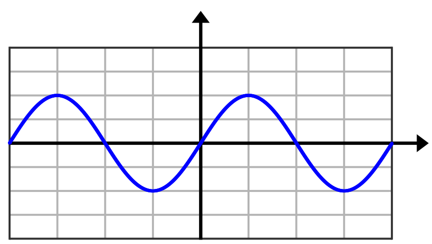
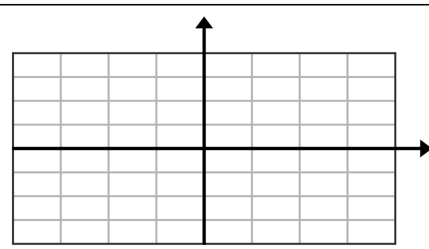
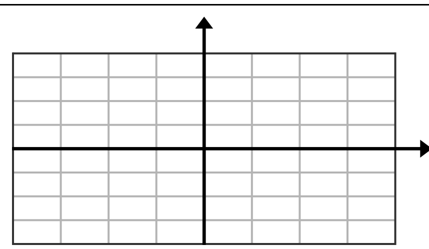
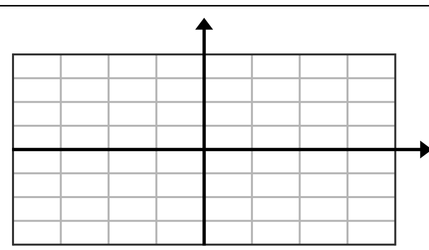
Investigation #4 Name ____________________

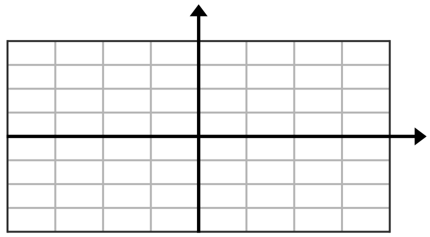
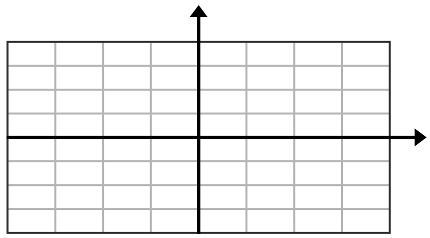
The Sine Function: Period

In this lesson you will learn how the constant B affects (or transforms) the graph of $y = \sin(Bx)$. Recall that the period of a sine graph is the length along the horizontal axis of one complete cycle.

1. Use a graphing calculator to graph each of the following functions. Our work will be done in degrees, so you must set the mode setting on your calculator to degrees. The suggested window settings are $X\min = -360$, $X\max = 360$, Xscl = 90, $Y\min = -2$, $Y\max = 2$, Yscl = 0.5. The first one has been done for you.

Be sure to identify the value of B for each graph.

Equation	B	Sketch	Number of Cycles in 360°	Period
$y = \sin(1x)$	1		1	360°
$y = \sin(2x)$				
$y = \sin\left(\frac{1}{2}x\right)$				
$y = \sin(4x)$				

Equation	B	Sketch	Number of Cycles in 360°	Period
$y=\sin\left(\frac{1}{4}x\right)$				
$y=\sin(8x)$				

2. Use the results of Exercise 1 to answer the following questions:
   a. If B = 1, the period of $y=\sin(Bx)$ is 360°. As B gets larger than 1, what happens to the period? ______________________________
   b. As B gets smaller than 1 (but still greater than 0) what happens to the period of the graph? ______________________________
   c. How does the number of cycles in 360° of a sine graph compare to the constant B? ______________________________
   d. Give a formula for the period of the sine function in terms of B. (Your formula must work for each graph in Exercise 1) ______________
3. If the graph of a sine wave shows 10 complete cycles in 360°, what is its period? ______________________________
4. Write an equation of the form $y=\sin(Bx)$ for each of the following periods.
   a. Period = 180° Equation: ____________________
   b. Period = 120° Equation: ____________________
   c. Period = 60° Equation: ____________________
5. Write an equation of the form $y=A\sin(Bx)+D$ whose graph is
   a. a sine curve with amplitude 2 and period 180°. ________________
   b. a sine curve with vertical shift -2 and period 90°. ________________
   c. a sine curve with amplitude 1.5, vertical shift 0.5 and period 720°.

   ____

6. Explain how the constant B affects (or transforms) the graph of $y=\sin(Bx)$.

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